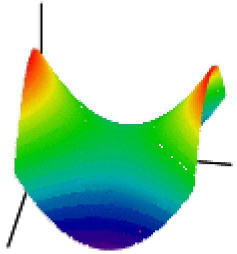
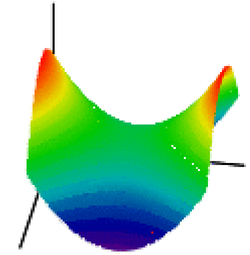


2.1 Current, Voltage and Resistance



- definition of charge and current
- what is voltage?
- origin of electrical resistance
- Ohm’s Law
- electrical properties of combined resistors
- creating an electrical potential
- measuring current and voltage with mechanical devices
- electronic voltage measurements

Charge and Current

• the unit of charge is the coulombs, C, where 1 C is the amount of charge that will deposit 0.0011180 g of Ag on a Pt electrode from a solution of ________

• the number of charges in a coulomb is determined by the average mass of Ag and Avogadro's number

$$\frac{0.0011180}{107.8682} \times 6.022137 \times 10^{23} = 6.2416 \times 10^{18}$$

• the reciprocal of this number is the charge on an electron

• the Faraday, $\mathcal{F}$, is the number of coulombs per mole, 96,485 C mol^{-1}

• current is the flow of charge, coulombs per second

• current has units of amperes, _________

• modern instrumentation can measure down to ________ (~6,000 electrons)

Voltage and Electron Repulsion

• voltage is a measure of the energy necessary to move charge, where _________

• the amount of power required to move charge is voltage times current

$$\text{V} \times \text{A} = \frac{\text{J}}{\text{C}} \times \frac{\text{C}}{\text{s}} = \frac{\text{J}}{\text{s}} = \text{W}$$

• electron-electron repulsion is responsible for requiring energy to move charge

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}}$$

where ε_0 is the permittivity of free space (vacuum), and r_{12} is the distance between the two charges, q_1 and q_2

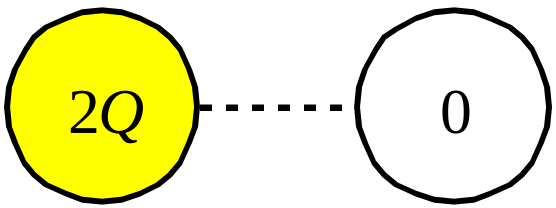
• permittivity is the ability of a material to ___________ ____________________, where $\varepsilon_0 = 8.8 \times 10^{-12}\ \text{C}^2\ \text{J}^{-1}\ \text{m}^{-1}$

Voltage between Two Spheres

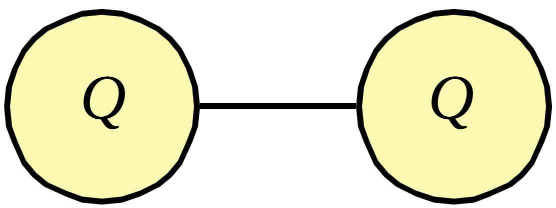
• energy required to place Q charges within a conducting sphere of radius a

$$U = \frac{3}{5}\frac{Q^2}{4\pi\varepsilon_0 a}$$

(5/3)a is interpreted as the average charge separation
• start with 2Q of charge in one sphere and compute energy released when the charge is equalized



$$U_{start} = \frac{3}{5}\frac{(2Q)^2}{4\pi\varepsilon_0 a} + 0 = \frac{12}{5}\frac{Q^2}{4\pi\varepsilon_0 a}$$



$$U_{end} = \frac{3}{5}\frac{Q^2}{4\pi\varepsilon_0 a} + \frac{3}{5}\frac{Q^2}{4\pi\varepsilon_0 a} = \frac{6}{5}\frac{Q^2}{4\pi\varepsilon_0 a}$$

Two Spheres (continued)

• the energy released by the transfer of charge is given by

$$\Delta U = U_{end} - U_{start} = -\frac{6}{5}\frac{Q^2}{4\pi\varepsilon_0 a}$$

• the same amount of energy would be needed to be provided to transfer all the charge back into the original sphere

• the required energy per charge can be expressed as a voltage,

$$V = \frac{\Delta U}{Q} = -\frac{6}{5}\frac{Q}{4\pi\varepsilon_0 a}$$

where the voltage is ________________________________

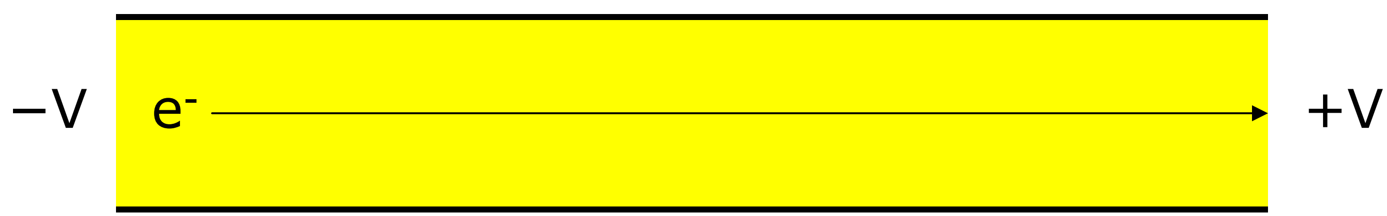
____.

• two regions of space containing different values of Q/a are characterized by an *electrical potential* (potential to do work)

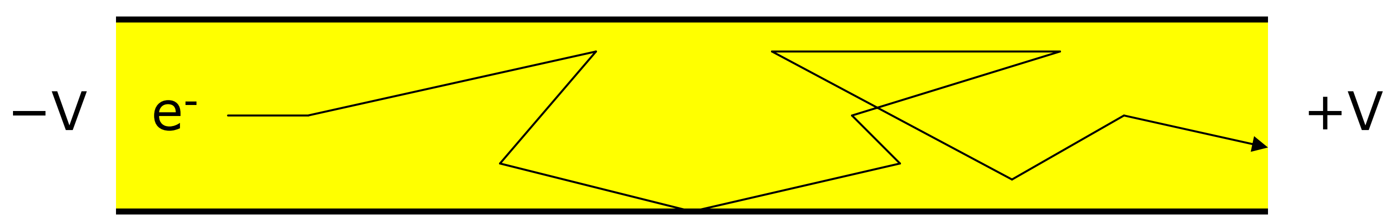
Physical Origin of Resistance

• in a metallic conductor the electrons exist in a partially filled conduction band and have access to the entire volume of the conductor

• in an ideal conductor the electron travels at ~___ m s^{-1}



• in a real conductor scatter off lattice imperfections and thermal motion of metal atoms reduce the net velocity to ~___ m s^{-1}



Derivation of Ohm's Law

• the density of current traveling through a conductor is given by the applied electric field, *E*, and the intrinsic material conductance, σ: $\mathcal{J} = \sigma E$

• current density is defined as $\mathcal{J} \equiv I/A$, while the electric field is defined as _______

• substitution of these two terms yields Ohm's Law

$$V = \left(\frac{l}{\sigma A}\right) I = \left(\frac{\rho l}{A}\right) I = RI$$

where ρ is the __________________ of the conductor

• R has units of Ohms, Ω, which in SI is J C^{-2} s, making ρ have units of Ω m

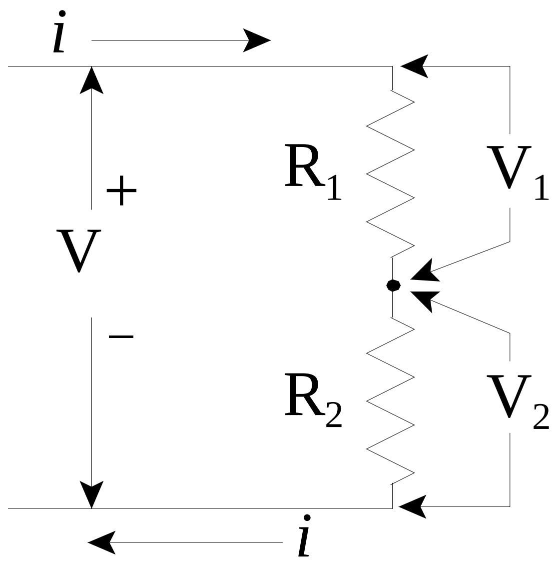
• R increases with resistor length and decreases with resistor area (A/l is a measure of the solid angle of scattered electrons collected at the end of the resistor)

Example Resistivities (Ω m)

silver	1.6×10^{-8}	
copper	1.7×10^{-8}	
nichrome	1.5×10^{-6}	
graphite	3.5×10^{-5}	crystalline value
silicon	640	semiconductor
quartz	7.5×10^{16}	insulator

- resistors made from compressed graphite particles have values from 1 Ω to 10 MΩ @ 5%
- wire-wound resistors vary in length and area to obtain values from 0.1 Ω to 1 MΩ @ 1%
- resistors made from a thin metal film, laser cut into a spiral, have values from 100 Ω to 100 kΩ @ 0.1%
- ceramic encased wire-wound resistors can handle up to 50 W

Series Wired Resistors



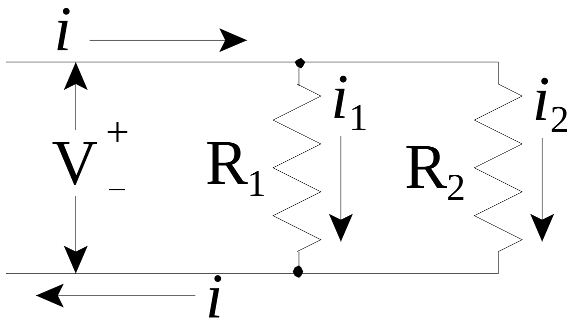
$$V = V_1 + V_2 = R_1 i + R_2 i = \left(R_1 + R_2\right) i = Ri$$

by convention *i* represents the flow of __________ charge

this circuit is called a voltage divider

$$V_1 = R_1 i = R_1 \frac{V}{R} = \frac{R_1}{R_1 + R_2} V \quad V_2 = R_2 i = R_2 \frac{V}{R} = \frac{R_2}{R_1 + R_2} V$$

Parallel Wired Resistors



$$i = i_1 + i_2 = \frac{V}{R_1} + \frac{V}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)V$$

let the voltage be equal to the effective resistance of the circuit times the current

$$i = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)iR$$

this circuit is called a current divider

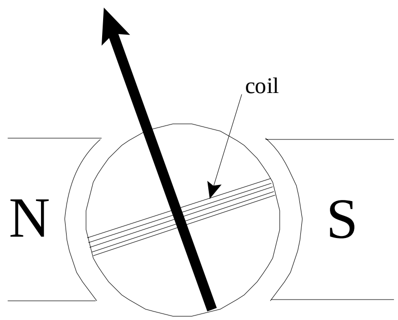
Creating an Electrical Potential

• an electrochemical cell generates an electric potential
• a battery consists of one or more electrochemical cells connected in series

lead-acid cell: $PbO_2 + Pb + 4H^+ + 2SO_4^{2-} \rightleftarrows 2PbSO_4 + 2H_2O$

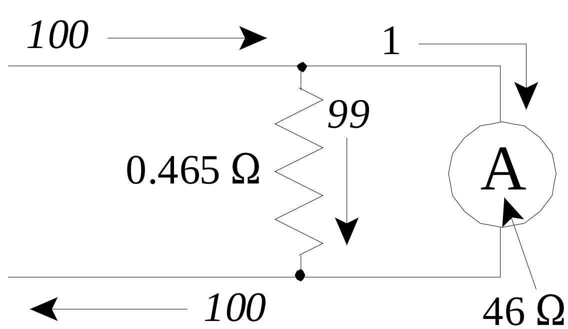
• the amount of voltage available depends upon the two half reactions - a 12 V car battery has six of these cells
• an electrical generator, or _________, is constructed by spinning a coil of conducting wire through a magnetic field
• the voltage is given by the velocity and strength of the field, where $F = qv \times B$
• high pressure steam is usually used to rotate the wire coils

Measuring Current Mechanically



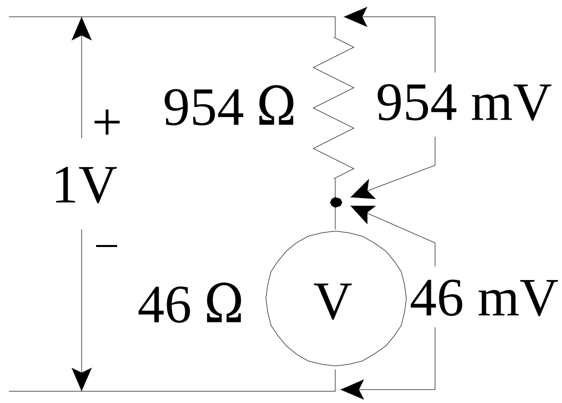
A wire coil carrying the current is placed in a magnetic field, the coil is attached to an axle, the force between the two magnetic fields causes the axle to rotate, rotation is hindered by a calibrated spring.

Depending upon the direction of the current the two fields repel or attract, rotating the axis in different directions.



A mechanical ammeter has a limited range of current over which it is linear (shown is 1 mA full scale deflection). The range is increased by using a "shunt" resistor to form a voltage divider with the coil resistance.

Measuring Voltage Mechanically



Since the ammeter coil has a resistance it can be used to measure voltage. Consider an ammeter with an internal resistance of 46 Ω and a full scale deflection for 1 mA. A series resistor is added so that the desired full scale voltage produces 1 mA of current. As an example, the above circuit has a total resistance of 1 kΩ, so that 1 V creates 1 mA of current.

The deflection scale is then relabeled in volts.

Measuring Voltage Electronically

- an unknown voltage is determined by comparison to a reference voltage

$$V_{std} - \alpha V_u = 0$$

the unknown voltage is amplified or attenuated by a variable amount, α, until the difference is zero

- the original standard was a Weston cell

$$Cd(Hg) \mid CdSO_4(aq), Hg_2SO_4(aq) \mid Hg$$

with an output of 1.018636 V @ 20 °C

- the current NIST standard is based on superconducting Josephson junctions (voltage-to-frequency conversion) accurate to 1 part in 10^9
- a practical reference voltage of 1.235 V can be obtained from transistor circuits that have almost no change with temperature