1. Isothermal: \( AT = 0 \) \( \Rightarrow \Delta U = \frac{3}{2} n R A T = 0 \)
   So, \( \Delta U = Q + W = 0 \) \( \Rightarrow Q = -W \)
   
   A) \( \frac{P_1}{P_2} \)
   \[
   W = -P dV = -P_2 (v_2 - v_1) \\
   = P_1 \left( \frac{n R T}{P_2} - \frac{n R T}{P_2} \right) \\
   = \text{nRT} \left( \frac{P_2}{P_1} - 1 \right) < 0
   
   \text{and} \ Q = -W = \text{nRT} \left( 1 - \frac{P_2}{P_1} \right) > 0
   
   B) \( \frac{P_1}{P_2} \)
   \[
   W = \int_{V_1}^{V_2} -P dV = \int_{V_1}^{V_2} \frac{n R T}{V} dV \\
   = -\text{nRT} \ln \left( \frac{V_2}{V_1} \right) < 0 \\
   
   \text{and} \ Q = -W = \text{nRT} \ln \left( \frac{V_2}{V_1} \right) > 0
   
2. Adiabatic: \( Q = 0 \) \( \Rightarrow \Delta U = W \)
   \[
   = U_2 - U_1 = \frac{3}{2} n R (T_2 - T_1)
   
   W = -P dV \\
   = -P_2 (v_2 - v_1) \\
   = -P_2 \left( \frac{n R T_2}{P_2} - \frac{n R T_1}{P_1} \right) = \Delta U
   
   \frac{3}{2} n R \left( T_2 - T_1 \right) = n R \left[ T_1 \left( \frac{P_2}{P_1} \right) - T_2 \right]
   
   T_2 \left( 1 + \frac{3}{2} \right) = T_1 \left( \frac{P_2}{P_1} \right) + \frac{3}{2} T_1 \\
   
   T_2 = \frac{2}{5} T_1 \left[ \frac{3}{2} + \frac{P_2}{P_1} \right]
   
   \text{two equations can be used to solve for } T_2 \text{ in terms of } T_1, P_1 \text{ and } P_2
3. Adiabatic: $Q = 0 \rightarrow \Delta U = W$

along the reversible adiabat

$U = \frac{3}{2} nRT$ and $P = \frac{nRT}{V}$

(the gas follows an equilibrium path)

at each small step along the way

$\frac{du}{dT} = \frac{3}{2} nRT$ and $dW_{\text{rev}} = -PdV$

so

$du = dW_{\text{rev}}$

$\frac{3}{2} nRdT = -\frac{nRT}{V} dV$ need to separate variables

and then integrate

$\frac{3}{2} \int \frac{nRdT}{T} = -\frac{nR}{V} \int dV$

$\frac{3}{2} \int \frac{T_2}{T_1} \frac{1}{T} dT = -\int \frac{V_2}{V_1} \frac{1}{V} dV$

$\frac{3}{2} \ln \left( \frac{T_2}{T_1} \right) = \ln \left( \frac{V_1}{V_2} \right)$

$\ln \left( \frac{T_2}{T_1} \right)^{\frac{3}{2}} = \ln \left( \frac{V_1}{V_2} \right)$

$\left( \frac{T_2}{T_1} \right)^{\frac{3}{2}} = \left( \frac{V_1}{V_2} \right)$

4. $T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\frac{2}{3}}$

$\frac{V_1}{V_2} = \frac{nRT_1}{P_1} = \frac{P_2 T_1}{P_1 T_2}$

$T_2 = T_1 \left( \frac{P_2 T_1}{P_1 T_2} \right)^{\frac{2}{3}}$

$T_2 \cdot T_2^{\frac{2}{3}} = T_1 \cdot T_1^{\frac{2}{3}} \left( \frac{P_2}{P_1} \right)^{\frac{2}{3}}$

$T_2^{\frac{5}{3}} = T_1^{\frac{5}{3}} \left( \frac{P_2}{P_1} \right)^{\frac{2}{3}}$

$T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{2}{5}}$
5. When a system does work adiabatically, the energy required to do the work must come from the system. So, if a weight is lifted in the surroundings, the energy of the system must decrease. If the system is an ideal monatomic gas then its energy is a function of temperature only (assuming the gas has a fixed number of moles). So, in order for the system to expend energy on lifting a weight, its temperature must go down. The actual final temperature obtained in such a process depends on how much weight was lifted. Since more weight is lifted when in a reversible process (as such a process is performed slowly and carefully enough so that no potential for doing work is wasted), the final temperature of the system must be lower when following a reversible adiabatic path than when following any other irreversible adiabatic path starting at the same state and ending at the same pressure.

6. In an isothermal expansion of an ideal gas system, the energy of the system is necessarily constant. So, any work that is done must be exactly compensated by an equal and opposite heat exchange. In other words, if a weight is lifted by the expanding system, then an equal amount of heat must be absorbed by the system. But where does this heat come from? The only possible answer is that it comes from the isothermal bath. Another way of looking at this is to recognize that in order to lift a weight the system must give up energy, which would tend to lower its temperature. However, since the system is in contact with an isothermal bath, any drop in system temperature will produce a heat transfer from the bath to the system, and this heat transfer will continue until the temperature of the system and bath have become equal.