Quantum Teleportation in Quantum Dots System

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Abstract

We present a model of quantum teleportation protocol based on one-dimensional quantum dots system. Three quantum dots with three electrons are used to perform teleportation, the unknown qubit is encoded using one electron spin on quantum dot A, the other two dots B and C are coupled to form a mixed space-spin entangled state. By choosing the Hamiltonian for the mixed space-spin entangled system, we can filter the space (spin) entanglement to obtain pure spin (space) entanglement, and after a Bell measurement, the unknown qubit is transfered to quantum dot B. Selecting an appropriate Hamiltonian for the quantum gate allows the spin-based information to be transformed into a charge-based information. The possibility of generalizing this model to N-electrons is discussed. The Hamiltonian to construct the CNOT gate, and the pulse sequence to realize the Hamiltonian are discussed in detail.

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I. INTRODUCTION

The special quantum features such as superpositions, interference and entanglement, have revolutionized the field of quantum information and quantum computation. Quantum teleportation primarily relies on quantum entanglement, which essentially implies an intriguing property that two quantum correlated systems can not be considered independent even if they are far apart. The dream of teleportation is to be able to travel by simply reappearing at some distant location. We have seen a familiar scene from science-fiction movies: The heroes shimmer out of existence to reappear on the surface of a faraway planet. This is the dream of teleportaion – the ability to travel from place to place without having to pass through the tedious intervening miles accompanied by a vehicle or an airplane. Although teleportation of large objects still remains a fantasy, quantum teleportaion has become a laboratory reality for photons, electrons and atoms[1–10].

By quantum teleportation an unknown quantum state is destroyed at a sending place while its perfect replica state appears at a remote place via dual quantum and classical channels. Quantum teleportation allows for the transmission of quantum information to a distant location despite the impossibility of measuring or broadcasting the information to be transmitted. The classical teleportation, like a fax, in which one could scan an object and send the information so that the object can be reconstructed at the destination. In this conventional facsimile transmission, the original object is scanned to extract partial information about it. The scanned information is then sent to the receiving station, where it is used to produce an approximate copy of the original object. The original object remains intact after the scanning process. By contrast, in quantum teleportation, the uncertainty principle forbids any scanning process from extracting all the information in a quantum state. The non-local property of quantum mechanics enables the striking phenomenon of quantum teleportation. Bennett and coworkers[11] showed that a quantum state can be teleported, providing one does not know that state, using a celebrated and paradoxical feature of quantum mechanics known as the Einstein-Podolsky-Rosen (EPR) effect[12]. They found a way to scan out part of the information from an object A, which one wishes to teleport, while causing the remaining part of the information to pass to an object B, via the EPR effect. In this process, two objects B and C form an entangled pair, object C is taken to the sending station, while object B is taken to the receiving station. At the sending station object C is scanned together with the original object A, yielding some information and totally disrupting the state of A and C. The scanned information is sent to the receiving station, where it is used to select one of several treatments to be applied to object B, thereby putting B into an exact replica of the former state of A.

Quantum teleportation exploits some of the most basic and unique features of quantum mechanics, teleportation of a quantum state encompasses the complete transfer of information from one particle to another. The complete specification of a quantum state of a system generally requires an infinite amount of information, even for simple two-level systems (qubits). Moreover, the principles of quantum mechanics dictate that any measurement on a system immediately alters its state, while yielding at most one bit of information. The transfer of a state from one system to another (by performing measurements on the first and operations on the second) might therefore appear impossible. However, it was shown that the property of entanglement in quantum mechanics, in combination with classical communication, can be used to teleport quantum states.

The application of quantum teleportation has been extended beyond the field of quantum communication. On one hand, quantum teleportation can be implemented using a quantum circuit that is much simpler than that required for any nontrivial quantum computational task: the state of an arbitrary qubit can be teleported using as few as two quantum CNOT gates. Thus, quantum teleportation is significantly easier to implement than even the simplest quantum computations if we are concerned only with the complexity of the required circuitry. On the other hand, quantum computing is meaningful even if it takes place very quickly and within a small region of space. The interest of quantum teleportation would be greatly reduced if the actual teleportation had to take place immediately after the required preparation. Quantum teleportation across significant time and space has been demonstrated with the technology that allows for the efficient long-term storage and purification of quantum information. Quantum teleportation of short-distance will play a role in transporting quantum information inside quantum computers. People have shown that a variety of quantum gates can be created by teleporting qubits through special entangled states[13,14]. This allows the construction of a quantum computer based on just single qubit operations, Bell's measurement, and the GHZ states. A wide variety of fault-tolerant quantum gates have also been constructed. Gottesman and Chuang demonstrated a procedure which performs an inner measurement conditioned on an outer cat state[13,14].

In quantum systems, interaction in general gives rise to entanglement. In this chapter, the entanglement in quantum dots system and its application for quantum teleportation will be discussed. We do not cover all the work that has been done in the field in this chapter. However, we chose a simple model to illustrate and introduce the subject. We present a model of quantum teleportation protocol based on one-dimensional quantum dots system. Three quantum dots with three electrons are used to perform teleportation: the unknown qubit is encoded using one electron spin on quantum dot A, the other two dots B and C are coupled to form a mixed space-spin entangled state. By choosing the Hamiltonian for the mixed space-spin entangled system, we can filter the space (spin) entanglement to obtain pure spin (space) entanglement and after a Bell measurement, the unknown qubit is transferred to quantum dot B. Selecting an appropriate Hamiltonian for the quantum gate allows the spin-based information to be transformed into the charge-based information. The possibility of generalizing this model to the N-electron syetem is discussed. The Hamiltonian to construct the CNOT gate will also be discussed in detail.

II. ENTANGLEMENT

Ever since the appearance of the famous Einstein, Podolsky and Rosen (EPR) experiment[12], the phenomenon of entanglement[15], which features the essential difference between classical and quantum physics[16], has received wide theoretical and experimental attention[16–23]. Generally speaking, if two particles are in an entangled state then, even if the particles are physically separated by a great distance, they behave in some respects as a single entity rather than as two separate entities. There is no doubt that entanglement has been lying in the heart of the foundation of quantum mechanics[24].

Besides quantum computations, entanglement has also been the core of many other active research such as quantum teleportation[25,26], dense coding[27,28], quantum communication[29] and quantum cryptography[30]. It is believed that the conceptual puzzles posed by entanglement – and discussed more than fifty years – have now become a physical source to brew completely novel ideas that might result in useful applications.

A big challenge faced by all the above-mentioned applications is to prepare the entangled states, which is much more subtle than classically correlated states. To prepare an entangled state of good quality is a preliminary condition for any successful experiment. In fact, this is not only a problem involved in experiments, but also pose an obstacle to theories since how to quantify entanglement is still unsettled, which is now becoming one of the central topics in quantum information theory. Any function that quantifies entanglement is called an entanglement measure. It should tell us how much entanglement there is in a given multipartite state. Unfortunately there is currently no consensus as to the best method to define entanglement for all possible multipartite states. The theory of entanglement is only partially developed[24,31–33] and can only be applied in a limited number of scenarios, where there is unambiguous way to construct suitable measures. Two important scenarios are (a) the case of a pure state of a bipartite system that is, a system consisting of only two components and (b) a mixed state of two spin-1/2 particles.

When a bipartite quantum system AB described by $H_A \otimes H_B$ is in a pure state there is an essentially well-motivated and unique measure of entanglement between the subsystems A and B given by the von Neumann entropy S. If we denote with ρ_A the partial trace of $\rho \in H_A \otimes H_B$ with respect to subsystem B, $\rho_A = Tr_B(\rho)$, the entropy of entanglement of the state ρ is defined as the von Neumann entropy of the reduced density operator ρ_A, $S(\rho) \equiv -Tr\left[\rho_A \log_2 \rho_A\right]$. It is possible to prove that, for pure states, the quantity S does not change if we exchange A and B. So we have $S(\rho) \equiv -Tr\left[\rho_A \log_2 \rho_A\right] \equiv -Tr\left[\rho_B \log_2 \rho_B\right]$. For any bipartite pure state, if the entanglement $E(\rho)$ is said to be good, it is often required to have the following properties: (1) For separable states ρ_{sep}, $E(\rho_{sep}) = 0$. (2) Reversible operations performed on the two subsystems A and B alone don't change the entanglement of the total systems. (3) The most general local operations that one can apply are non-unitary. (4) The last property for a good measure of entanglement is that if we take two bipartite systems in the total state $\rho_t = \rho_1 \otimes \rho_2$, we should have $E(\rho_t) = E(\rho_1) + E(\rho_2)$. It is possible to show that the quantity S has all the above properties. Clearly, S is not the only mathematical object that meet the requirement (1)-(4), but in fact, it is accepted as the correct and unique measure of entanglement.

Generally, the strict definitions of the four most prominent entanglement measures can be summarized as follows[35]: (1) Entanglement of distillation E_D; (2) Entanglement of cost E_C; (3) Entanglement of formation E_F and finally (4) Relative entropy of entanglement E_R. The first two measures are also called operational measures while the second two do not admit a direct operational interpretation in terms of entanglement manipulations. It can be proved that, suppose E is a measure defined on mixed states which satisfy the conditions for a good entanglement measure mentioned above, then for all states $\rho \in (H^A \otimes H^B)$, $E_D(\rho) \leq E(\rho) \leq E_C(\rho)$, and both $E_D(\rho)$ and $E_C(\rho)$ coincides on pure states with the von Neumann reduced entropy as having been demonstrated above. For the fermion system, we chose to use Zanardi's measure[36], which is given in Fock space as the von Neumann entropy.

III. QUANTUM TELEPORTATION

Quantum teleportation is an entanglement-assisted teleportation. It is a technique used to transfer information on a quantum level, usually from one particle (or series of particles) to another particle (or series of particles) in another location via quantum entanglement. Its distinguishing feature is that it can transmit the information present in a quantum superposition, which is useful for quantum communication and computation.

More precisely, quantum teleportation is a quantum protocol by which the information on a qubit A (quantum bit, a two-level quantum system) is transmitted exactly (in principle) to another qubit B. This protocol requires a conventional communication channel capable of transmitting two classical bits, and an entangled pair (B, C) of qubits, with C at the origin location with A and B at the destination. The protocol has three steps: measure A and C jointly to yield two classical bits; transmit the two bits to the other end of the channel; and use the two bits to select one of four ways of recovering B.

The two parties are Alice (A) and Bob (B), and a qubit is, in general, a superposition of quantum state $|0\rangle$ and $|1\rangle$. Equivalently, a qubit is a unit vector in two-dimensional Hilbert space. Suppose Alice has a qubit in some arbitrary quantum state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. Assume that this quantum state is not known to Alice and she would like to send this state to Bob. A solution to this problem was discovered by Bennet et al.[11]. The parts of a maximally entangled two-qubit state are distributed to Alice and Bob. The protocol then involves Alice and Bob interacting locally with the qubits in their possession and Alice sending two classical bits to Bob. In the end, the qubit in Bob's possession will be transformed into the desired state.

Alice and Bob share a pair of entangled qubits BC. That is, Alice has one half, C, and Bob has the other half, B. Let A denote the qubit Alice wishes to transmit to Bob. Alice applies a unitary operation on the qubits AC and measures the result to obtain two classical bits. In this process, the two qubits are destroyed. Bob's qubit, B, now contains information about C; however, the information is somewhat randomized. More specifically, Bob's qubit B is in one of four states uniformly chosen at random and Bob cannot obtain any information about C from his qubit. Alice provides her two measured qubits, which indicate which of the four states Bob possesses. Bob applies a unitary transformation which depends on the qubits he obtains from Alice, transforming his qubit into an identical copy of the qubit C.

Suppose the qubit A that Alice wants to teleport to Bob can be written generally as: $|\psi\rangle_A = \alpha|0\rangle + \beta|1\rangle$. Alice and Bob to share a maximally entangled state beforehand, for instance one of the four Bell states:

$$\begin{aligned}
|\Phi^+\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_C \otimes |0\rangle_B + |1\rangle_C \otimes |1\rangle_B) \\
|\Phi^-\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_C \otimes |0\rangle_B - |1\rangle_C \otimes |1\rangle_B) \\
|\Psi^+\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_C \otimes |1\rangle_B + |1\rangle_C \otimes |0\rangle_B) \\
|\Psi^-\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_C \otimes |1\rangle_B - |1\rangle_C \otimes |0\rangle_B). \qquad (1)
\end{aligned}$$

Alice takes one of the particles in the pair, and Bob keeps the other one. The subscripts C and B in the entangled state refer to Alice's or Bob's particle. We will assume that Alice and Bob share the entangled state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_C \otimes |0\rangle_B + |1\rangle_C \otimes |1\rangle_B)$. So, Alice has two particles (A, the one she wants to teleport, and C, one of the entangled pair), and Bob has one particle, B. In the total system, the state of these three particles is given by

$$|\psi\rangle_A \frac{1}{\sqrt{2}}(|0\rangle_C|0\rangle_B + |1\rangle_C|1\rangle_B) \tag{2}$$

where subscripts A and C are used to denote Alice's system, and subscript B to denote Bob's system. This three particle state can be rewritten in the Bell basis as:

$$\frac{1}{2}(|\Phi^+\rangle(\alpha|0\rangle + \beta|1\rangle) + |\Phi^-\rangle(\alpha|0\rangle - \beta|1\rangle) + |\Psi^+\rangle(\beta|0\rangle + \alpha|1\rangle) + |\Psi^-\rangle(-\beta|0\rangle + \alpha|1\rangle)) \tag{3}$$

The teleportation starts when Alice measures her two qubits in the Bell basis. Given the above expression, the results of her measurement is that the three-particle state would collapse to one of the following four states (with equal probability of obtaining each)

$$\begin{aligned} &|\Phi^+\rangle(\alpha|0\rangle + \beta|1\rangle) \\ &|\Phi^-\rangle(\alpha|0\rangle - \beta|1\rangle) \\ &|\Psi^+\rangle(\beta|0\rangle + \alpha|1\rangle) \\ &|\Psi^-\rangle(-\beta|0\rangle + \alpha|1\rangle). \end{aligned} \tag{4}$$

Alice's two particles are now entangled to each other, in one of the four Bell states. The entanglement originally shared between Alice's and Bob's qubits is now broken. Bob's particle takes on one of the four superposition states shown above. Bob's qubit is now in a state that resembles the state to be teleported. The four possible states for Bob's qubit are unitary images of the state to be teleported.

The local measurement done by Alice on the Bell basis gives complete knowledge of the state of the three particles; the result of her Bell measurement tells her which of the four states the system is in. She simply has to send her results to Bob through a classical channel. Two classical bits can communicate which of the four results she obtained.

After Bob receives the message from Alice, he will know which of the four states his particle is in. Using this information, he can rotate the target qubit into the correct state $|\psi\rangle$ by applying the appropriate unitary transformation I, σ_Z, σ_X or $i\sigma_Y$. Quantum teleportation using pairs of entangled photons[38–43] and atoms[8,9] have been demonstrated experimentally. There are also schemes suggesting using electrons to perform quantum teleportation[4,7,44].

IV. ENTANGLEMENT IN THE ONE-DIMENSIONAL HUBBARD MODEL

Quantum dots system is one of the proposals for building a quantum computer[45,46]. With dimensions ranging from a mere 1 nm to as much as 100 nm and consisting of anywhere between 10^3 to 10^6 atoms and electrons, semiconductor quantum dots are often regarded as artificial atoms. Charge carriers in semiconductor quantum dot are confined in all three dimensions and the confinement can be achieved through electrical gating and/or etching techniques applied to a two-dimensional electron gas. To describe the quantum dots, a simple approximation is to regard each dot as having one valence orbital, the electron occupation could be $|0>$, $|\uparrow>$, $|\downarrow>$ and $|\uparrow\downarrow>$, with other electrons treated as core electrons[47]. The valence electron can tunnel from a given dot to its nearest neighbor obeying the Pauli principle and thereby two dots can be coupled together, this is the electron hopping effect. Another effect needs to be considered is the on-site electron-electron repulsion. A theoretical description of an array of quantum dots can be modelled by the one-dimensional Hubbard Hamiltonian:

$$H = -t \sum_{<ij>,\sigma} c_{i\sigma}^{\dagger}\, c_{j\sigma} + U \sum_{i} n_{i\uparrow}\, n_{i\downarrow} \tag{5}$$

where t stands for the electron hopping parameter, U is the Coulomb repulsion parameter for electrons on the same site, i and j are the neighboring site numbers, $c_{i\sigma}^{\dagger}$ and $c_{j\sigma}$ are the creation and annihilation operators.

Entanglement using Zanardi's measure can be formulated as the von Neumann entropy given by

$$E_j = -Tr(\rho_j log_2 \rho_j), \tag{6}$$

where the reduced density matrix ρ_j is given by

$$\rho_j = Tr_j(|\Psi><\Psi|), \tag{7}$$

Tr_j denotes the trace over all but the jth site and $|\Psi>$ is the antisymmetric wave function of the fermion system. Hence E_j actually describes the entanglement of the j-th site with the remaining sites.

In the Hubbard model, the electron occupation of each site has four possibilities, there are four possible local states at each site, $|\nu>_j = |0>_j$, $|\uparrow>_j$, $|\downarrow>_j$, $|\uparrow\downarrow>_j$. Since the Hamiltonian is invariant under translation, the local density matrix ρ_j of the j-th site is site independent and is given by[48]

$$\rho_j = z|0\rangle\langle 0| + u^+|\uparrow\rangle\langle\uparrow| + u^-|\downarrow\rangle\langle\downarrow| + w|\uparrow\downarrow\rangle\langle\uparrow\downarrow| \tag{8}$$

with

$$w = \langle n_{j\uparrow} n_{j\downarrow}\rangle = Tr(n_{j\uparrow} n_{j\downarrow}\rho_j) \tag{9}$$

$$u^+ = \langle n_{j\uparrow}\rangle - w, \qquad u^- = \langle n_{j\downarrow}\rangle - w \tag{10}$$

$$z = 1 - u^+ - u^- - w = 1 - \langle n_{j\uparrow}\rangle - \langle n_{j\downarrow}\rangle + w \tag{11}$$

The Hubbard Hamiltonian can be re-scaled to have only one parameter U/t. The entanglement of the j-th site with the other sites is given by[48]

$$E_j = -zLog_2 z - u^+ Log_2 u^+ - u^- Log_2 u^- - wLog_2 w. \tag{12}$$

For the one-dimensional Hubbard model with half-filled electrons, we have $< n_\uparrow >=< n_\downarrow >= \frac{1}{2}$, $u^+ = u^- = \frac{1}{2} - w$, and the local entanglement is given by

$$E_j = -2wlog_2 w - 2(\frac{1}{2} - w)log_2(\frac{1}{2} - w) \tag{13}$$

For each site the entanglement is the same. Consider the particle-hole symmetry of the model, we can see that $w(-U) = \frac{1}{2} - w(U)$, so the local entanglement is an even function of U. As shown in Fig. 1, the minimum of the entanglement is 1 as $U \to \pm\infty$. As $U \to +\infty$ all the sites are singly occupied the only difference is the spin of the electrons on each site, which can be referred as the spin entanglement. As $U \to -\infty$, all the sites are either doubly occupied or empty, which is referred as the space entanglement. The maximum entanglement is 2 at $U = 0$, which is the sum of the spin and space entanglement of the system. In Fig. 1, we show the entanglement for two sites and two electrons, they qualitatively agree with that of the Bethe ansatz solution for an array of sites[48].

V. QUANTUM TELEPORTATION IN QUANTUM DOTS

Gittings and Fisher[49] showed that the entanglement in this system can be used in quantum teleportation. However, in their scheme both the charge and spin of the system are used to construct the unitary transformation. Here, we describe a different scheme to perform quantum teleportation. For two half-filled coupled quantum dots, under the conservation of the total number of electrons $N = 2$ and the total electron spin $S = 0$, a quantum entanglement of 2, two ebits (if each of the entangled particles is used to encode a qubit, the entangled joint states is called an ebit or entangled bit. Ebits are "shared resources" that require both particles) can be produced according to Zanardi's measure. Let us describe the teleportation scheme using three sites, A, B and C. Suppose the qubit $|\Psi >= \alpha| \uparrow> +\beta| \downarrow>$ will be teleported from site A (Alice), to site B (Bob) where the two sites B and C are in an entangled state,

$$|\Psi_{CB} >= \frac{1}{\sqrt{2}}(\ c_{C\uparrow}^{\dagger}+ \ c_{B\uparrow}^{\dagger})\frac{1}{\sqrt{2}}(\ c_{C\downarrow}^{\dagger}+ \ c_{B\downarrow}^{\dagger})|0> . \tag{14}$$

A spin-up electron and a spin-down electron are in a delocalized state on sites C and B. In the occupation number basis $|\ n_{C\uparrow}\ n_{C\downarrow}\ n_{B\uparrow}\ n_{B\downarrow}>$, the state of the system can be written as:

$$|\Psi_{CB} >= \frac{1}{\sqrt{2}}(\ c_{C\uparrow}^{\dagger}+ \ c_{B\uparrow}^{+})\frac{1}{\sqrt{2}}(\ c_{C\downarrow}^{\dagger}+ \ c_{B\downarrow}^{\dagger})|0> \frac{1}{2}(|0011> +|1100> +|1001> +|0110>). \tag{15}$$

From the state described by Eq. (10) we can see that in the basis of $|\ n_{C\uparrow}\ n_{C\downarrow}>$, there are four possible states: $|00>$, $|11>$, $|10>$, $|01>$. Corresponding to each of the states on site C, the states on site B are: $|11>$, $|00>$, $|01>$, $|10>$ in the occupation number basis $|\ n_{B\uparrow}\ n_{B\downarrow}>$. Under the restriction of the conservation of total number of electrons and total spin of the system, two ebits can be obtained, one is in the spatial degree of freedom, and the other is in the spin degree of freedom. In the basis of $|\ n_{C\uparrow}\ n_{C\downarrow}\ n_{B\uparrow}\ n_{B\downarrow}>$, the two ebits are:

$$\beta_0 = \frac{1}{\sqrt{2}}(|1100> +|0011>), \qquad \beta_1 = \frac{1}{\sqrt{2}}(|1001> +|0110>) \tag{16}$$

These two ebits can be used in quantum teleportation. The C-NOT operation in the occupation number basis $|\ n_{A\uparrow}\ n_{A\downarrow}\ n_{C\uparrow}\ n_{C\downarrow}>$ is given by:

$$|1000>\leftrightarrow |1011>, |1010>\leftrightarrow |1001>, |01\ n_{C\uparrow}\ n_{C\downarrow}>\leftrightarrow |01\ n_{C\uparrow}\ n_{C\downarrow}> \tag{17}$$

For the ebit β_0, in the quantum teleportation process, in basis $|\ n_{A\uparrow}\ n_{A\downarrow}>|\ n_{C\uparrow}\ n_{C\downarrow}\ n_{B\uparrow}\ n_{B\downarrow}>$, as shown in Fig. 2, we have the initial state in the quantum dots:

$$|\Psi_0 >= (\alpha|10> +\beta|01>)\frac{1}{2}(|1100> +|0011> +|1001> +|0110>) \tag{18}$$

Alice performs the CNOT operation on the two qubits she holds, using the source qubit as a control qubit and the half EPR qubit as target qubit:

$$|\Psi_1>=\alpha|10>\frac{1}{\sqrt{2}}(|0000>+|1111>)+\beta|01>\frac{1}{\sqrt{2}}(|1100>+|0011>) \tag{19}$$

she performs the Hadamard operation on the initial qubit:

$$|\Psi_2>=\alpha(|10>+|01>)\frac{1}{2}(|0000>+|1111>)+\beta(|10>-|01>)\frac{1}{2}(|1100>+|0011>) \tag{20}$$

After these operations, Alice does the measurement M_1 and M_2 on the two qubits she holds, the following results will be obtained:

$$\begin{array}{ll} |M_1M_2> & |\, n_{B\uparrow}\, n_{B\downarrow}> \\ |1011> & \alpha|11>+\beta|00> \\ |1000> & \alpha|00>+\beta|11> \\ |0111> & \alpha|11>-\beta|00> \\ |0100> & \alpha|00>-\beta|11> \end{array} \tag{21}$$

Then after performing a unitary transformation using double electron occupation and zero electron occupation as basis, the source qubit can be obtained on site B. For this system the Hamiltonian to perform the C-NOT operation is given by:

$$\begin{aligned} H_{CNOT} &= |10>_{A\,A}<10|(|11>_{C\,C}<00|+|00>_{C\,C}<11|)+ \\ & |01>_{A\,A}<01|(|11>_{C\,C}<11|+|00>_{C\,C}<00|) \\ & =\frac{1}{2}(\sigma_Z^A+1)(\, c_{C\uparrow}^{\dagger}\, c_{C\downarrow}^{\dagger}+c_{C\uparrow}\, c_{C\downarrow})+ \\ & \frac{1}{2}(1-\sigma_Z^A)(\, c_{C\uparrow}^{\dagger}\, c_{C\downarrow}^{\dagger}\, c_{C\uparrow}\, c_{C\downarrow}+c_{C\uparrow}\, c_{C\downarrow}\, c_{C\uparrow}^{\dagger}\, c_{C\downarrow}^{\dagger}), \end{aligned} \tag{22}$$

where σ_Z^A is the Pauli matrix. We can see that by using this Hamiltonian, the spin entanglement of the system is filtered, the space entanglement is used in the teleportation process. An important result is that the original state we want to teleport is in a superposition state of spin up and spin down electrons. However, after the teleportation process, the state we obtained on site B is a superposition state of double electron occupation and zero electron occupation. The information based on spin has been transformed to information based on charge, but the information content is not changed. It is well known that a difficult task in quantum information processing and spintronics is the measurement of a single electron spin[50], in the scheme above, we changed the quantum information from spin-based to charge-based, thus makes the measurement fairly easier. This is also important in quantum computation based on electron spin since the readout can be easily measured.

The Hamiltonian for the C-NOT operation can be realized by constructing pulse sequences using the tools of geometric algebra. The tools of geometric algebra provide a useful means of constructing pulse sequences for quantum logic operations[51]. The method is based on the use of primitive idempotents. The primitive idempotents, $E_{\pm}$, satisfy the following properties:

$$E_+ + E_- = 1, \qquad (E_{\pm})^2 = E_{\pm}, \qquad E_+E_- = 0. \tag{23}$$

These idempotents can help simplify exponential operations as follows:

$$e^{A \cdot E_{\pm}} = e^A E_{\pm} + E_{\mp}, \qquad\qquad (if \quad [A, E_{\pm}] = 0). \tag{24}$$

For spin-$\frac{1}{2}$ particles, the idempotents of interest are

$$E^i_{\pm} = \frac{1}{2}(1 \pm \sigma^i_Z), \qquad\qquad E^{i,j}_{\pm} = \frac{1}{2}(1 \pm \sigma^i_Z \sigma^j_Z), \tag{25}$$

where σ's are the Pauli matrices. E^A_+ is thus the density matrix for the A spin in the up state and E^B_- is the density matrix for the B spin in the down state. Such operators have been useful in other NMR quantum computing experiments[52].

$$E_+ = |0\rangle\langle 0| \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{26}$$

$$E_- = |1\rangle\langle 1| \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{27}$$

$$\sigma_x E_+ = |1\rangle\langle 0| \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_x E_- = |0\rangle\langle 1| \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{28}$$

Using the definitions of E_+, E_- and σ_x, the Hamiltonian for C-NOT gate can be rewritten in a simpler form. In this part we transform the state representation from Fock space to the standard quantum computing representation: $|10\rangle = |\uparrow\rangle = |0\rangle$, $|01\rangle = |\downarrow\rangle = |1\rangle$. In the entangled pair we define $|11\rangle_C = |\uparrow\downarrow\rangle = |0\rangle$ and $|00\rangle_C = |\emptyset\rangle = |1\rangle$. Then the Hamiltonian can be written as:

$$H_{CNOT} = E_+^A(\sigma_X^C E_-^C + \sigma_X^C E_+^C) + E_-^A(E_-^C + E_+^C) = E_+^A \sigma_X^C + E_-^A \tag{29}$$

The physical interpretation of the above equation is an instruction to perform the σ_X operation on site C if site A is spin-up and to perform the identity operation if the state on site A is spin-down.

The expression of the problem in terms of idempotents also makes the generation of the pulse sequence quite straightforward. The propagator for the C-NOT can be factorized into elements which can be physically applied. This is accomplished by first rewriting the propagator as

$$H_{CNOT} = E_-^A + E_+^A \sigma_X^C = E_-^A + (i)(-i)E_+^A \sigma_X^C \tag{30}$$

which can be factorized into

$$H_{CNOT} = (E_-^A - iE_+^A \sigma_X^C)(E_-^A + iE_+^A) \tag{31}$$

Using the fact that the idempotents can be expressed as exponentials, the above expression becomes:

$$H_{CNOT} = e^{-iE_+^A \sigma_X^C \pi/2} \cdot e^{iE_+^A \pi/2} \tag{32}$$

This expression can be expressed as

$$H_{CNOT} = e^{i\pi/4} \cdot e^{-i\sigma_X^C \pi/4} \cdot e^{-i\sigma_Z^A \pi/4} \cdot e^{i\sigma_Z^A \sigma_X^C \pi/4} \tag{33}$$

This is an exact expression for the propagator, and is also the pulse sequence for its implementation. Note here the basis for σ_X^C is different from the basis for σ_X^A, the basis for the former is double and zero occupation of site C, and the basis for the later is spin-up and spin-down state, so in the operation σ_X^C will transform between state $|11\rangle$ and $|00\rangle$ in Fock space.

For another ebit β_1, in the quantum teleportation process, in basis $| n_{A\uparrow}\ n_{A\downarrow} > | n_{C\uparrow}\ n_{C\downarrow}\ n_{B\uparrow}\ n_{B\downarrow} >$, we have:

$$|\Psi_0 >= (\alpha|10 > +\beta|01 >)\frac{1}{2}(1100 > +|0011 > +|1001 > +|0110 >) \tag{34}$$

$$|\Psi_1 >= \alpha|10 > \frac{1}{\sqrt{2}}(|0101 > +|1010 >) + \beta|01 > \frac{1}{\sqrt{2}}(|1001 > +|0110 >) \tag{35}$$

$$|\Psi_2>=\alpha(|10>+|01>)\frac{1}{2}(|0101>+|1010>+\beta(|10>-|01>)\frac{1}{2}(|1001>+|0110>) \tag{36}$$

When Alice does the measurement M_1 and M_2, the following results will be obtained:

$$\begin{array}{ll} |M_1M_2> & |\ n_{B\uparrow}\ n_{B\downarrow}> \\ |1001> & \alpha|01>+\beta|10> \\ |1010> & \alpha|10>+\beta|01> \\ |0101> & \alpha|01>-\beta|10> \\ |0110> & \alpha|10>-\beta|01> \end{array} \tag{37}$$

For this system the Hamiltonian to perform the C-NOT operation is:

$$\begin{aligned} H_{CNOT} &= |10>_{A\ A}<10|(|10>_{C\ C}<01|+|01>_{C\ C}<10|)+ \\ &\quad |01>_{A\ A}<01|(|01>_{C\ C}<01|+|10>_{C\ C}<10|) \\ &= \frac{1}{2}(\sigma_Z^A+1)(\ c_{C\uparrow}^{\dagger}\ c_{C\downarrow}+\ c_{C\downarrow}^{\dagger}\ c_{C\uparrow}) \\ &\quad +\frac{1}{2}(1-\sigma_Z^A)(\ c_{C\uparrow}^{\dagger}\ c_{C\uparrow}\ c_{C\downarrow}\ c_{C\downarrow}^{\dagger}+\ c_{C\downarrow}^{\dagger}\ c_{C\downarrow}\ c_{C\uparrow}\ c_{C\uparrow}^{\dagger}) \end{aligned} \tag{38}$$

Then after doing a unitary transformation using the electron spin up and spin down as basis, the source qubit can be recovered on site B. By using this Hamiltonian for the C-NOT operation the space entanglement of the system is filtered, the spin entanglement is used in the process.

Using the geometric techniques of idempotents, the Hamiltonian for the C-NOT gate can be written in a simpler form. Here we transform the representation of the qubit state from Fock space to standard quantum computing state: $|10\rangle=|\uparrow\rangle=|0\rangle$, $|01\rangle=|\downarrow\rangle=|1\rangle$. In the entangled pair we define $|10\rangle_C=|\uparrow\rangle=|0\rangle$ and $|01\rangle_C=|\downarrow\rangle=|1\rangle$. Then the Hamiltonian can be rewritten as:

$$H_{CNOT}=E_+^A(\sigma_X^C E_-^C+\sigma_X^C E_+^C)+E_-^A(E_-^C+E_+^C)=E_-^A+E_+^A\sigma_X^C \tag{39}$$

The physical interpretation of the above equation is an instruction to perform the σ_X operation of site C if site A is spin-up and to do the identity operation if the site A is spin-down.

The propagator for the C-NOT operation can be constructed as follows, first rewriting the propagator as:

$$H_{CNOT}=E_-^A+E_+^A\sigma_X^C=E_-^A+(i)(-i)E_+^A\sigma_X^C \tag{40}$$

which can be factorized into

$$H_{CNOT} = (E_-^A - iE_+^A\sigma_X^C)(E_-^A + iE_+^A) \tag{41}$$

Using the fact that the idempotents can be expressed as exponentials, the above expression becomes:

$$H_{CNOT} = e^{-iE_+^A\sigma_X^C\pi/2} \cdot e^{iE_+^A\pi/2} \tag{42}$$

This expression can be expressed as:

$$H_{CNOT} = e^{i\pi/4} \cdot e^{-i\sigma_X^C\pi/4} \cdot e^{-i\sigma_Z^A\pi/4} \cdot e^{i\sigma_Z^A\sigma_X^C\pi/4} \tag{43}$$

This is an exact expression for the propagatorand is also a pulse sequence for its implementation. Note here the basis for σ_X^C is the same as for σ_X^A, the electron spin-up and spin-down state.

For $U \neq 0$, the state of the 2-electron 2-sites system can be described as follows:

$$|\Psi>= a_1|1100> + a_2|0011> + b_1|1001> + b_2|0110>; \;\; a_1^2 + a_2^2 + b_1^2 + b_2^2 = 1, \tag{44}$$

where $a_1 = a_2$, $b_1 = b_2$ because of the symmetry in the entangled pairs, such that the state can be written as:

$$|\Psi>= a\beta_0 + b\beta_1; \;\; a^2 + b^2 = 1. \tag{45}$$

From the above analysis, we can see that in the case of using β_0 or β_1 as ebits, the unitary transformation is performed in the occupation number basis of $| \; n_{B\uparrow} \; n_{B\downarrow} >$, using basis $|11>, |00>$ or $|10>, |01>$. We can select the basis separately, either charge or spin. We can also choose the Hamiltonian (one is related to the spin entanglement and the other is related to space entanglement) for the CNOT operation, when the Hamiltonian for one ebit is chosen, the ebit corresponding to the other Hamiltonian is filtered.

If $U > 0$, the contribution of the spin entanglement to the total entanglement is greater than that of the space entanglement. The probability of getting the ebit $|\beta_1>$ increases as U becomes larger. If $U < 0$, the contribution of the space entanglement to the total entanglement becomes greater than that of the spin entanglement, the probability of getting the ebit $|\beta_0>$ increases as U becomes more negative. In the limit of U goes to $\pm\infty$, only spin entanglement or space entanglement will exist. This might be related to the spin charge separation in the Hubbard model[53]. In a previous study[54], we showed that the maximum entanglement can be reached at $U > 0$ by introducing asymmetric electron hopping impurity to the system. This is very convenient in the quantum information processing. We can control the parameter U/t to increase the probability of getting either ebit.

VI. SUMMARY

We have proposed two schemes for teleportation of a single qubit in quantum dots system modelled by the one-dimensional Hubbard Hamiltonian, two ebits are contained in the system and can be used in the teleportation process. Now we analyze the theoretical fidelity of these two teleportation schemes. The fidelity of teleportation is defined as the projection of the teleported state $|\psi'\rangle$ on site C to the initial state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ on the site A, $|\langle\psi|\psi'\rangle|^2$. If Alice can distinguish all four possible measurement outcomes, the teleportation process can, in principle, be completed with a 100% success rate and is deterministic. If Alice, on the other hand, is only able to perform a partial measurement on her two particles, the success probability is less than 100% and the teleportation is probabilistic. In the first scheme, when the space entangled ebit is used, Alice does the measurement in charge basis. She can only distinguish on site C, whether it is doubly charged or has no charge. As a result, she can only distinguish two measurement results, thus the fidelity of this scheme is 50%. In the second scheme, by using the spin entanglement, Alice does the measurement in spin basis, all four measurement results can be distinguished, thus the fidelity is 100%.

We discussed implementing quantum teleportation in three-electron system. For more electrons and in the limit of $U \to +\infty$ there is no double occupation, the system is reduced to the Heisenberg model, in the magnetic field. The neighboring spins will favor the anti-parallel configuration for the ground state. If the spin at one end is flipped, then the spins on the whole chain will be flipped accordingly due to the spin-spin correlation. Such that the spins at the two ends of the chain are entangled, a spin entanglement, this can be used for quantum teleportation, the information can be transferred through the chain. For $U \neq +\infty$, for the N-sites N-electron system with $S = 0$, the first $N - 1$ sites entangled with the N-th site in the same way as that of the two-electron two-sites system: if the N-th site has 2 electrons, then the first $N-1$ sites will have $N-2$ electrons; if the N-th site has 0 electrons, then the first $N-1$ sites will have N electrons; if the N-th site has 1 spin-up electron, then the total spin of the first $N-1$ sites will be 1 spin-down; if the N-th site has the 1 spin-down electron, then the total spin of the first $N-1$ sites will be 1 spin-up. So the same procedure discussed above can be used for quantum teleportation but the new system with N-electrons is much more complicated than the previous three electron system. Moreover, Alice needs to control the first $N-1$ sites and the source qubit. This situation is different from the spin chain. The correlation can not be transferred from one end to the other.

We have studied the entanglement of an array of quantum dots modelled by the one-dimensional Hubbard Hamiltonian and its application in quantum teleportation. The entanglement in this system is a mixture of space and spin entanglement. The application of such entanglement in quantum teleportation process has been discussed. By applying different Hamiltonian for the CNOT operation, we can separate the ebit based on space entanglement or spin entanglement and apply it in quantum teleportation process. It turns out that if we use the ebit of the space entanglement, we can transform the spin-based quantum information to the charge-based quantum information making the measurement fairly easy.

Efficient long-distance quantum teleportation is crucial for quantum communication and quantum networking schemes. Ursin[55] et. al have performed a high-fidelity teleportation of photons over a distance of 600 meters across the River Danube in Vienna, with the optimal efficiency that can be achieved using linear optics. Another exciting experiment in quantum communication has also been done by Ursin et. al[56,57]. One photon is measured locally at the Canary Island of La Palma, whereas the other is sent over an optical free-space link to Tenerife, where the Optical Ground Station of the European Space Agency acts as the receiver. This exceeds previous free-space experiments by more than an order of magnitude in distance, and is an essential step towards future satellite-based quantum communication. Recently decoy-state quantum cryptography over a distance of 144 km between two Canary Islands was demonstrated successfully. Such experiments also open up the possibility of quantum communication on a large scale using satellites.

Teleportation of single qubits is insufficient for a large-scale realization of quantum communication and quantum computation. Many scientists have developed and exploited teleportation of two-qubit composite system using a six-photon interferometer[58]. In this experiment, a six-photon interferometer has been exploited to teleport an arbitrary polarization state of two photons. The observed teleportation fidelities for different initial states are all well beyond the state estimation limit of 0.40 Not only does six-photon interferometer provide an important step towards teleportation of a complex system, it will also enable future experimental investigations on a number of fundamental quantum communication and computation protocols.

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[1] R. L. de Visser and M. Blaauboer. Deterministic Teleportation of Electrons in a Quantum Dot Nanostructure. *Physical Review Letters*, 96(24):246801, 2006.

[2] Jian-Wei Pan, Dik Bouwmeester, Matthew Daniell, Harald Weinfurter and Anton Zeilinger. Experimental test of quantum nonlocality in three-photon Greenberger-Horne-Zeilinger entanglement. *Nature*, 403(3):515, 2000.

[3] P. Chen, C. Piermarocchi, and L. J. Sham. Control of exciton dynamics in nanodots for quantum operations. *Physical Review Letters*, 87(6):067401, 2001.

[4] F. de Pasquale, G. Giorgi, and S. Paganelli. Teleportation on a quantum dot array. *Physical Review Letters*, 93(12):12052, 2004.

[5] J. H. Reina and N. F. Johnson. Quantum teleportation in a solid-state system. *Physical Review A*, 63(1):012303, 2000.

[6] D. Bouwmeester, J. Pan, K. Mattle, M. Eibl, H. Weinfurter and A. Zeilinger. Experimental quantum teleportation. *Nature*, 390(11):575, 1997.

[7] C. W. J. Beenakker and M. Kindermann. Quantum teleportation by particle-hole annihilation in the Fermi sea. *Physical Review Letters*, 92(5):056801, 2004.

[8] M. D. Barrett, J. Chiaverinl, T. Schaetz, J. Britton, W. M. Itano, J. D. Jost, E. Knill, C. Langer, D. Leibfried, R. Ozeri and D. J. Wineland. Deterministic quantum teleportation of atomic qubits. *Nature*, 429(17):737, 2004.

[9] M. Riebe, H. Häffner, C. F. Roos, W. Hänsel, J. Benhelm, G. P. T. Lancaster, T. W. Körber, C. Becher, F. Schimdt-Kaler, D. F. V. James and R. Blatt. Deterministic quantum teleportation with atoms. *Nature*, 429(17):734, 2004.

[10] H. Wang, S. Kais, Quantum teleportation in one-dimensional quantum dots system. *Chem. Phys. Lett.*, 421: 338, 2006.

[11] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).

[12] A. Einstein, B. Podolsky and N. Rosen. Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?, *Phys. Rev.* **47**, 777 (1935).

[13] G. Brassard, S. L. Braunstein and R. Cleve. Teleportation as a quantum computation. *Physica*

D, 120(1):43, 1998.

[14] D. Gottesman and I. Chuang, Demonstrating the Viability of Universal Quantum Computation Using Teleportation and Single-Qubit Operations, *Nature*, 402:390-393, 1999.

[15] E. Schrödinger. Discussion of Probability Relations Between Separated Systems, *Proc. Cambridge Phil. Soc.*, **31**, 555 (1935).

[16] J. S. Bell. On the Einstein-Podolsky-Rosen Paradox, *Physics*, **1**, 195-200 (1964).

[17] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters. Mixed-state entanglement and quantum error correction, *Phys. Rev.*, **A 54**, 3824 (1996).

[18] E. Hagley, X. Maytre, G. Nogues, C. Wunderlich, M. Brune, J. M. Raimond, and S. Haroche. Generation of Einstein-Podolsky-Rosen Pairs of Atoms, *Phys. Rev. Lett.*, **79**, 1(1997).

[19] Q. A. Turchette, C. S. Wood, B. E. King, C. J. Myatt, D. Leibfried, W. M. Itano, C. Monroe, and D. J. Wineland. Deterministic Entanglement of Two Trapped Ions, *Phys. Rev. Lett.*, **81**, 3631 (1998).

[20] Dik Bouwmeester, Jian-Wei Pan, Matthew Daniell, Harald Weinfurter, and Anton Zeilinger. Observation of Three-Photon Greenberger-Horne-Zeilinger Entanglement, *Phys. Rev. Lett.*, **82**, 1345-1349 (1999).

[21] C. Monroe, D. M. Meekhof, B. E. King, and D. J. Wineland. A Schrödinger cat superposition state of an atom, *Science*, **272**, 1131 (1996).

[22] C.A. Sackett, D. Kielpinksi, B.E. King, C. Langer, V. Meyer, C. J. Myatt, M. Rowe, Q.A.Turchette, W .M. Itano, D. J. Wineland and C. Monroe. *Nature*, **404**, 256-259 (2000).

[23] A. Peres. *Quantum Theory: Concepts and Methods* Kluwer Academic Publishers, Boston 1995.

[24] G. Vidal, W. Dur, JI Cirac. Entanglement cost of bipartite mixed states, *Phys. Rev. Lett.*, **89**, 027901 (2002).

[25] D. Bouwmeester, K. Mattle, J.-W. Pan, H. Weinfurter, A. Zeilinger, and M. Zukowski. Experimental quantum teleportation of arbitrary quantum states, *Appl. Phys.*, **67**, 749 (1998).

[26] D. Boyuwmeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter and A. Zeilinger. *Nature*, **390**, 575 (1997).

[27] C. H. Bennett and S. J. Wiesner. Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states, *Phys. Rev. Lett.*, **69**, 2881 (1992).

[28] Mattle K., Harald Weinfurter, Paul G. Kwiat and Anton Zeilinger. *Phys. Rev. Lett.*, **76**, 4546 (1996).

[29] B. Schumacher. Quantum coding, *Phys. Rev. A*, **51**, 2738 (1995).

[30] C. H. Bennett, G. Brassard, C. Creau, R. Jozsa, A. Peres, and W. K. Wootters. Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels, *Phys. Rev. Lett.*, **70**, 1895 (1993).

[31] M. Horodecki, P. Horodecki, and R. Horodecki. Asymptotic manipulations of entanglement can exhibit genuine irreversibility, *Phys. Rev. Lett.*, **86**, 5844-5844 (2001).

[32] M. Horodecki, P. Horodecki, and R. Horodecki. Separability of n-particle mixed states: necessary and sufficient conditions in terms of linear maps, *Phys. Lett. A*, **283** 1-7 (2001).

[33] W. K. Wooters. Parallel transport in an entangled ring, *J. Math. Phys.*, **43**, 4307-4325 (2002).

[34] M. A. Nielsen, C. M. Dawson, J. L. Dodd, A. Gilchrist, D. Mortimer, T. J. Osborne, M. J. Bremner, A. W. Harrow and A. Hines, Phys. Rev. A **67**, 052301 (2003).

[35] S. Kais, "Reduced-Density-matrix Mechanics with Applications to Many-electron Atoms and Molecules", Advances in Chemical Physics Vol 134, pp 493, Edited by D.A. Mazziotti (Wiley, New York, 2007).

[36] P. Zanardi, Phys. Rev. A **65**, 042101 (2002).

[37] M.A. Nielson, E. Knill and R. Laflamme, "Complete quantum teleportation using nuclear magnetic resonance", Nature **396**, 52 (1998).

[38] D. Bouwmeester, J. Pan, K. Mattle, M. Eibl, H. Weinfurter and A. Zeilinger, Nature **390**, 575 (1997).

[39] D. Boschi, S. Branca, F. DeMartini, L. Hardy and S. Popescu, Phys. Rev. Lett. **80**, 1121 (1998).

[40] J. Pan, M. Daniell, S. Gasparoni, G. Weihs and A. Zeilinger, Phys. Rev. Lett. **86**, 4435 (2001).

[41] I. Marcikic, H. Riedmatten, W. Tittel, H. Zbinden and N. Gisin, Nature **421**, 509 (2003).

[42] D. Fattal, E. Diamanti, K. Inoue and Y. Yamamoto, Phys. Rev. Lett. **92**, 037904 (2004).

[43] A. Furusawa, J. Sorensen, Science **282**, 706 (1998).

[44] O. Sauret, D. Feinberg, and T. Martin, Eur. Phys. J. B **32**, 545 (2003).

[45] D. Loss, D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).

[46] M. Friesen, M. P. Rugheimer, D. Savage, M. Lagally, D. van der Weide, R. Joynt, Phys. Rev. B **67**, 121301 (2003).

[47] F. Remacle and R. D. Levine, PNAS, **97**, 553 (2000).

[48] S. Gu, S. Deng, Y. Li, and H. Lin, Phys. Rev. Lett. **93**, 086402 (2004).

[49] J. R. Gittings and A. J. Fisher, Phys. Rev. A **66**, 032305 (2002).

[50] R. Ionicioiu and A.E. Popescu, arXiv:quant-ph/0310047 v2, (2005).

[51] M. D. Price, S. S. Somaroo, C. H. Tseng, J. C. Gore, A. F. Fahmy, T. F. Havel, and D. G. Cory, J. Magn. Res., 140: 371-378 (1999).

[52] Z. L. Madi, R. Bruschweiler, and R. R. Ernst, J. Chem. Phys., 109: 10603-10611 (1998).

[53] F. H. L. Essler, H. Frahm, F. G¨ohmann, A. Kl¨umper and V. E. Korepin, *The one-dimensional Hubbard model* (Cambridge, 2005).

[54] H. Wang and S. Kais, Entanglement and Quantum Phase Transition in a One-Dimensional System of Quantum Dots with Disorders, International Journal of Quantum Information. 4(5): 827 (2006).

[55] R. Ursin, T. Jennewein, M. Aspelmeyer, R. Kaltenbaek, M. Lindenthal, P. Walther and A. Zeilinger. Quantum teleportation across the Danube. *Nature*, 430: 849, 2004.

[56] R. Ursin, F. Tiefenbacher, T. Schmitt-Manderbach, H. Weier, T. Scheidl, M. Lindenthal, B. Blauensteiner, T. Jennewein, J. Perdiguess, P. Trojek, B. Ömr, M. Fürst, M. Meyenburg, J. Rarity, Z. Sodnik, C. Barbieri, H. Weinfurter and A. Zeilinger. Entanglement-based quantum communication over 144km. *Nature Physics*, 3(6): 481, 2007.

[57] Markus Aspelmeyer, Thomas Jennewein, Martin Pfennigbauer, Walter R. Leeb, and Anton Zeilinger. Long-Distance Quantum Communication With Entangled Photons Using Satellites, *IEEE Journal of Selectied Topics in Quantum Electronics*, 9(6), 2003.

[58] Q. Zhang, A. Goebel, C. Wagenknecht, Y. Chen, B. Zhao, T. Yang, A. Mair, J. Schmiedmayer, and J. Pan. Experimental quantum teleportation of a two-qubit composite system. *Nature Physics*, 2(10):678, 2006.

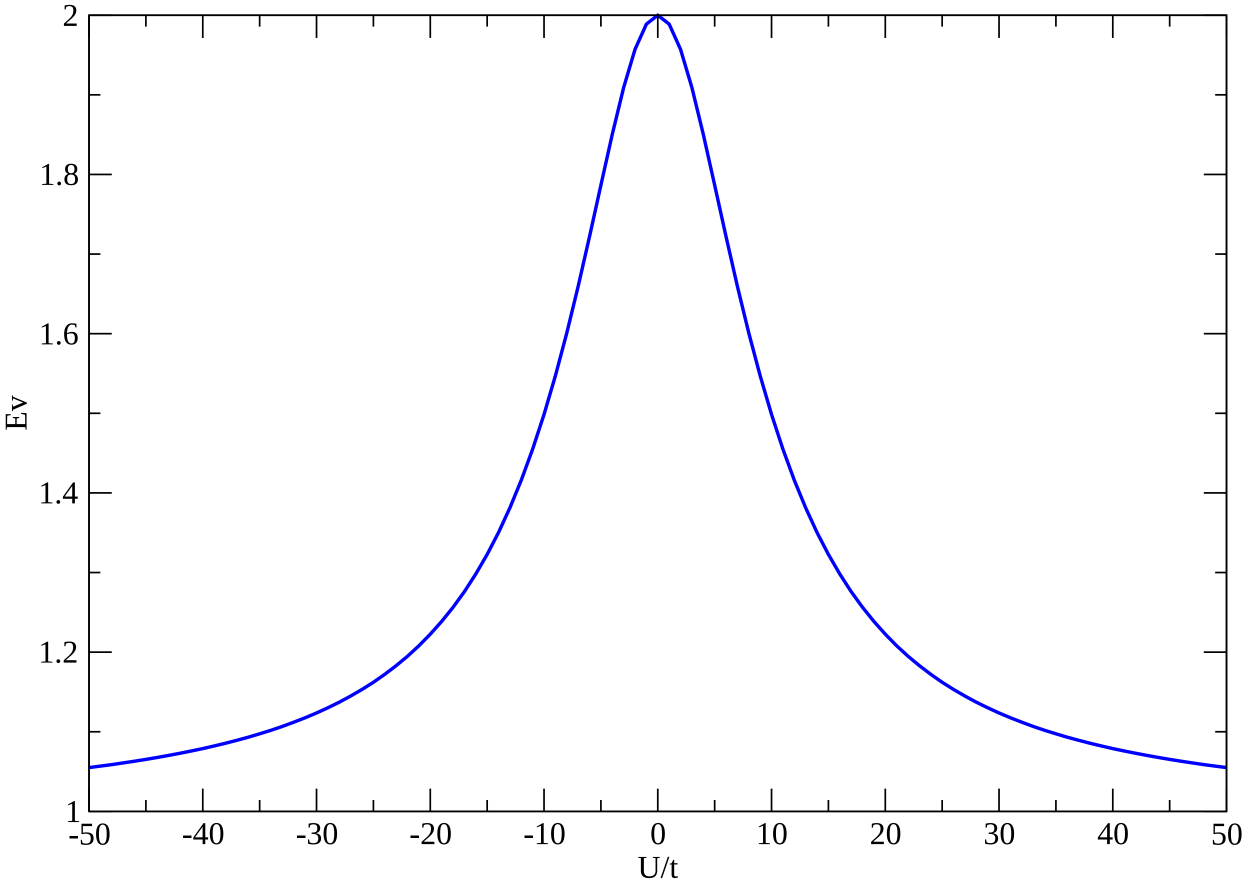

FIG. 1: Local entanglement given by the von Neumann entropy E_v versus U/t for two sites two electrons.

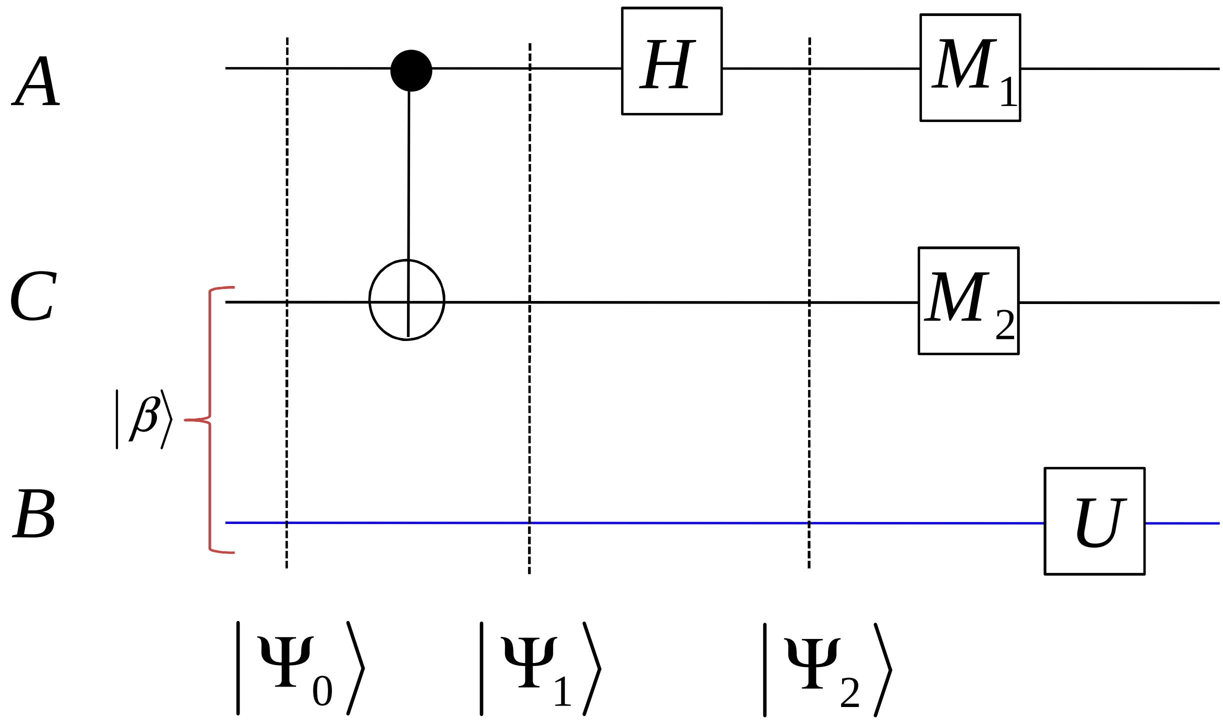

FIG. 2: Quantum circuit for teleporting a qubit. The two top lines represent Alice's system, while the bottom line is Bob's system. β is an entangled pair of qubits Alice and Bob share. H represents a Hadamard transformation, M_1 and M_2 represent the measurement on the two top lines. U represents a unitary operation that Bob performs to rotate his qubit to the state Alice teleport. $|\Psi_0\rangle$ is the initial state for the whole system, $|\Psi_1\rangle$ is the state after Alice performs CNOT operation, and $|\Psi_2\rangle$ is the state after Alice performs Hadamard operation on initial qubit she holds; The outcome is the teleported state Bob will get after performing a unitary operation according to the result of the measurement Alice made.