Representational Fluency in Learning and Problem Solving in Physics

N. Sanjay Rebello
Elizabeth Gire

Physics Education Research Group

Department of Physics
Kansas State University

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Dr. Elizabeth Gire
Asst. Professor, Univ. of Memphis*

Adrian Carmichael
Jacquelyn Chini
Dong-Hai Nguyen

Graduate Students

* Starts August 2010
Collaborators

- Sadhana Puntambekar
  - Dept. of Educ. Psychology, University of Wisconsin – Madison

- Andrew G. Bennett
  - Dept. of Mathematics, Kansas State University

- David H. Jonassen
  - Dept. of Educ. Psychology, University of Missouri – Columbia
What is Representational Fluency?

“The ability to comprehend the equivalence of different modes of representation” (Sigel & Cocking, 1977)

“Comprehend Equivalence“:
- Read out info presented in different representations.
- Transform information from one representation to other.
- Learn in one representation and apply to other.
- Others...

“Modes of Representation”:
- Verbal vs. Mathematical
- Graphical vs. Equational
- Macroscopic vs. Microscopic
- Physical vs. Virtual
- Others...

Representational Fluency involves Transfer
Some Views of Transfer

- Identical elements must exist between situations.
- Knowledge must be encoded in a coherent model.
- Students either transfer or they don’t.
- Researchers/educators pre-decide what must transfer.
- Static one-shot assessment e.g. tests and exams.
- Focus mainly on students’ internal knowledge.

Transfer is rare.

E.g. Gick & Holyoak (1980), Reed & Ernst (1974), Thorndike (1906)
Some Emerging Views of Transfer

- (Re) construct knowledge in new context.
- Knowledge can transfer in pieces.
- Learners may transfer some pieces, but not others.
- We must examine anything that transfers.
- Dynamic, real-time assessment e.g. interviews.
- Focus also on mediating factors e.g. motivation.

Transfer is ubiquitous.

Hammer et al (2005), diSessa & Wagner (2005);
Our View of Transfer

Transfer is the creation of **associations** between new information and prior knowledge.

The association is **controlled** by other factors e.g. learners’ epistemology, motivation, emotions, etc.

Redish (2004)
Two Kinds of Associations

- Assigning a new case to an existing knowledge element.
  - e.g. The electric field between two parallel plates is constant.

- Constructing an association between two knowledge elements.
  - e.g. Integral of Electric field is the Electric potential.
Two Kinds of Transfer

‘Horizontal’
- Activating and mapping a pre-constructed model to a new situation.
- Associations between read-out information of a situation & elements of model.

‘Vertical’
- Constructing a new model to make sense of a situation.
- Association between knowledge elements to create model.
Our Framework of Transfer

- **‘Vertical’ Transfer**
  - Constructing or Re-constructing a model to make sense of new information
  - Mapping of new information onto existing model

- **‘Horizontal’ Transfer**
  - Existing model
### Alignment with Others’ Views

<table>
<thead>
<tr>
<th><strong>Horizontal</strong></th>
<th><strong>Vertical</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Assimilation</td>
<td>Accommodation 1</td>
</tr>
<tr>
<td>Efficiency</td>
<td>Innovation 2</td>
</tr>
<tr>
<td>Model Development</td>
<td>Model Deployment 3</td>
</tr>
<tr>
<td>Class C Transfer</td>
<td>Class A Transfer 4</td>
</tr>
<tr>
<td>Low Road Transfer</td>
<td>High Road Transfer 5</td>
</tr>
<tr>
<td>Applicative knowledge</td>
<td>Interpretive knowledge 6</td>
</tr>
<tr>
<td>Sequestered Problem Solving</td>
<td>Preparation for Future Learning 7</td>
</tr>
<tr>
<td>Used in structured, traditional contexts, which involves few internal representations activated repeatedly</td>
<td>Used in ill-structured, non-traditional contexts, which involves choosing, or constructing multiple internal representations 8</td>
</tr>
</tbody>
</table>

1 Piaget (1952)  
2 Schwartz, Bransford & Sears (2005)  
3 Hestenes (1987)  
4 diSessa & Wagner (2005)  
6 Broudy (1977)  
7 Bransford & Schwartz (1999)  
8 Jonassen (2003)
What Transfer do We Want?

Horizontal (Efficiency) AND Vertical (Innovation)

Striking a Balance: ‘Optimal Adaptability Corridor’

1 Schwartz, Bransford & Sears (2005)

2 Murray & Arroyo (2002)
Some Caveats

‘Horizontal’ & ‘Vertical’ Transfer...

- are not mutually exclusive.
  - A given thinking process might involve elements of **both** ‘horizontal’ and ‘vertical’ transfer.

- cannot be universally labeled.
  - What is perceived as ‘**vertical**’ transfer by a novice may be perceived as ‘**horizontal**’ transfer by an expert.
Possible Research Questions (RQs)

- How do students engage in ‘horizontal’ and ‘vertical’ transfer?
- Under what conditions do they engage in each?
- Is there a preferred sequence for these processes?
- and several others....
RQs For this Talk...

How does the sequence in which learners interact with different representations affect

- learning?
  - **Study 1**: Learning using Physical vs. Virtual Representations

- problem solving?
  - **Study 2**: Solving Problems in Numerical vs. Graphical vs. Equational Representations
Study 1: Background

- Previous studies -- mixed results
  - Virtual outperform analogous physical experiments
    - Zacharia, Olympiou, & Papaevripidou, 2008
    - Finkelstein, et al., 2005
  - No difference in learning: physical vs. virtual
    - Klahr, Triona, & Williams, 2007
    - Zacharia & Constantinou, 2008

- Zacharia & Constantinou (2008)
  - More research is needed to describe how physical and virtual manipulatives should be integrated in a curriculum.
Study 1: Research Questions

When students use both physical & virtual representation...

- How does their learning from the two representations compare?
- How does the sequence of using the physical and virtual representations affect students’ learning?
Study 1: Research Context

- **CoMPASS Curriculum** (Puntambekar et al, 2003)
  - Concept Mapped Project-based Activity Scaffolding System
  - Integrates: Hypertext + Activities (Physical/Virtual)

- **Pulley Unit**: Two-hour lab
  - Targeted models:

[Diagram of pulley system with force needed and work done relationships]
Study 1: Physical & Virtual Representations

Pulley Simulation

- **Pulley System**
  - Single Fixed
  - Two Fixed
  - Single Movable
  - Single Compound
  - Double Compound
  - Triple Compound

- **Experiment Set Up**
  - Load: 4.9 N
  - Distance to Lift: 0.1 m

- **Controls**
  - Effort Force: 1.225 N

- **Measurements**
  - Distance Pulled: 0.4 m
  - Distance Moved: 0.1 m
  - Work Done: 0.49 J
Study 1: Research Design

PV Sequence (N=61)
- Physical-Virtual Sequence

Pre-Test
- Physical-Experiment
- 13 multiple-choice conceptual questions
- Cornbach Reliability ~ 0.75

Mid-Test
- Physical-Experiment
- Make predictions
- Set up various pulley systems
  - For each ...
    - Measure Force needed
    - Measure Distance pulled
    - Calculate Work & PE
    - Discuss trends across systems
- Choose various pulley systems
  - For each ...
    - Observe Force needed
    - Observe Distance pulled
    - Observe Work & PE
- Discuss trends across systems

VP Sequence (N=71)
- Virtual-Physical Sequence

Post-Test
- Virtual-Experiment
- Physical-Experiment
Study 1: Overall Test Performance

Repeated Measures ANOVA Score x Sequence Interaction p-value ~0.001 (Mid-Post)
Study 1: ‘Force’ Questions on Test

Repeated Measures ANOVA
Score x Sequence Interaction
p-value ~0.02 (Pre-Mid)
Study 1: ‘Work’ Questions on Test

% CORRECT

PRE | MID | POST

PHYSICAL-VIRTUAL (N=71)
VIRTUAL-PHYSICAL (N=61)
Study 1: Why these Results?

Two possible effects: Differential

- Cue salience?  
  (Denton & Kruschke, 2006)

- Ambiguous Data?  
  (Chinn & Brewer, 1993)
Study 1: What Causes Differential Cue Salience?

- Superiority / Noticing effect? (Lindgren & Schwartz, 2009)

Pulley Simulation

- 'Force' & 'Work' Equally Salient
- 'Force' Salient
- But 'Work' = ?
Overshadowing? (e.g. Heckler, et al 2006)

Salience is high in both: Learning from whichever cue is presented first: Primacy effect

Increasing Salience: Learning

Decreasing Salience: Overshadowing
### Study 1: Ambiguous Data

- **Data that is learner** (Chinn & Brewer, 1993)...
  - ambiguous may be ignored by the learner
  - Unambiguous may facilitate learning
- **Ambiguity due to**: measurement error, friction, etc.
- In our case, for student data on ‘Work’

<table>
<thead>
<tr>
<th>Type of Pulley System</th>
<th>Work value determined in PHYSICAL experiment</th>
<th>Work value measured in VIRTUAL experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Fixed</td>
<td>.49 J</td>
<td>.50 J</td>
</tr>
<tr>
<td>Double Compound</td>
<td>.52 J</td>
<td>.50 J</td>
</tr>
<tr>
<td></td>
<td>.48 J</td>
<td>.50 J</td>
</tr>
<tr>
<td></td>
<td>.53 J</td>
<td>.50 J</td>
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</tbody>
</table>

**Physical**: Ambiguous → Does not promote learning

**Virtual**: Unambiguous → Promotes learning
## Study 1: Implication of Differential Ambiguity

### Physical vs. Virtual Cues

<table>
<thead>
<tr>
<th></th>
<th>Physical P</th>
<th>Virtual V</th>
</tr>
</thead>
<tbody>
<tr>
<td>‘Force’ Data</td>
<td>Unambiguous</td>
<td>Unambiguous</td>
</tr>
<tr>
<td>‘Work’ Data</td>
<td>Ambiguous</td>
<td>Unambiguous</td>
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</tbody>
</table>

### Virtual vs. Physical Cues

<table>
<thead>
<tr>
<th></th>
<th>Virtual V</th>
<th>Physical P</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>‘Work’ Cues</td>
<td>Unambiguous</td>
<td>Ambiguous</td>
</tr>
</tbody>
</table>

- **Increasing Unambiguity:** Learning
- **Decreasing Unambiguity:** No Learning
Study 1: Horizontal & Vertical transfer...

Physical

Virtual

Initial model → After Prediction → After Experiment

Initial model → After Prediction

After Experiment
Study 1: Horizontal & Vertical transfer...

Physical-Virtual Sequence

Virtual-Physical Sequence

Physical Virtual

Virtual Physical

No new learning occurs in Physical Activity
Study 1: Horizontal & Vertical transfer...

No new learning occurs
Study 1: Conclusions

When students use both physical & virtual representations...

- Overall, if physical is used first, students continue to learn when virtual is used afterward, but not vice versa.

- Effect of sequencing varies with the concept being learned:
  - 'Force': Learned most from whatever presented first (Primacy effect)
  - 'Work': Better learned from virtual rather than physical (Overshadowing, Ambiguity in Data)

If they don’t learn anything more from physical after doing virtual, then why do both, just do virtual?
RQs For this Talk...

How does the sequence in which learners interact with different representations affect

- learning?
  - **Study 1:** Learning using Physical vs. Virtual Representations

- problem solving?
  - **Study 2:** Solving Problems in Numerical vs. Graphical vs. Equational Representations
Study 2: Motivation

Multiple Representations (MRs) useful in solving physics problems

- Several studies addressing the benefits of using MRs in solving physics problems.

- Not as many studies on how students transfer their problem solving skills in physics across different MRs.
RQ2.1: What difficulties do students encounter when transferring their problem solving processes across multiple representations?

RQ2.2: How do those difficulties change which the sequence in which these representations are presented?
Study 2: Research Context

- N=20 participants
- Engineering majors
- Enrolled in 1st semester calc-based physics
- Topics: Kinematics, Work-Energy
Study 2: Research Methodology

Data Collection: Teaching/Learning Interviews
(Steffe et al., 2003)
- Four sessions: One after each class exam
- Each session: 60 minutes, video/audio taped
- Three problems per session
- Hints provided when students expressed difficulties

Data Analysis: Phenomenographic coding (Marton, 1986)
- Coded, categorized difficulties expressed by student
- Inter-rater reliability ~ 0.8
Study 2: Research Design

EG Sequence (N=10)
- Equation-Graph Sequence

GE Sequence (N=10)
- Graph – Equation Sequence

Original (Verbal) Problem
  - Equational Problem
  - Graphical Problem

Equational Problem
  - Graphical Problem

Graphical Problem
Example: Original Problem (Verbal)

A hoop radius \( r = 1 \) cm and mass \( m = 2 \) kg is rolling at an initial speed \( v_i \) of 10 m/s along a track as shown. It hits a curved section (radius \( R = 2.0 \) m) and is launched vertically at point A.

What is the launch speed of the hoop as it leaves the curve at point A?
Example: Graphical Problem

A sphere radius \( r = 1 \text{ cm} \) and mass \( m = 2 \text{ kg} \) is rolling at an initial speed \( v_i \) of 5 m/s along a track as shown. It hits a curved section (radius \( R = 1.0 \text{ m} \)) and is launched vertically at point A. The rolling friction on the straight section is negligible. The magnitude of the rolling friction force acting on the sphere varies as angle \( \theta \) as per the graph shown.

What is the launch speed of the hoop as it leaves the curve at point A?
Example: Equational Problem

A sphere radius \( r = 1 \text{ cm} \) and mass \( m = 2 \text{ kg} \) is rolling at an initial speed \( v_i \) of 5 m/s along a track as shown. It hits a curved section (radius \( R = 1.0 \text{ m} \)) and is launched vertically at point A. The rolling friction on the straight section is negligible. The magnitude of the rolling friction force acting on the sphere varies as angle \( \theta \) (radians) as per the equation shown.

\[
F_{roll}(\theta) = -0.7\theta^2 - 1.2\theta + 4.5
\]

What is the launch speed of the hoop as it leaves the curve at point A?
Case Reuse (Jonassen, 2006)
- Tried to mimic the previous problems
  - Example: Attempting to find work done by friction by multiplying force with distance.

Graphical Interpretation
- Instinctively tried to calculate the slope of graph
- Several hints to recognize integral is area under graph

Physical Interpretation of Math Procedures
- Adequate knowledge of math procedures
- Inability to apply these procedures in physics problems
- Hints on reflecting on units of physical quantities effective
Study 2: Results - Sequencing Effect

- **Equation-Graph** sequence may cause more difficulties to students than the **Graph-Equation** sequence*

* Not statistically significant
Study 2: Toy Model of Difficulty

Contributions

\[ D_{\text{Total}} = D_{\Delta R} + D_{\Delta C} + D_{\Delta O} \]

- Total # of Difficulties
- Difficulties due to Change in Representation
- Difficulties due to Change in Context
- Difficulties due to all Other Changes
Study 2: Results - Sequencing Effect

- Most difficulties are due to change in representation ($D_{VR}^R$).
- Decline in $D_{VR}^R$ in going from 2nd problem to 3rd problem regardless of sequence.
- $D_{VR}^R [\text{Verbal} \rightarrow \text{Equation}] > D_{VR}^R [\text{Verbal} \rightarrow \text{Graph}]^*$

*Not statistically significant.
Study 2: Results - Sequencing Effect

Difficulties Due to Change in Context

<table>
<thead>
<tr>
<th>Problem Encountered</th>
<th>Average # Difficulties</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td></td>
</tr>
<tr>
<td>P2</td>
<td></td>
</tr>
<tr>
<td>P3</td>
<td></td>
</tr>
</tbody>
</table>

Difficulties Due to Other Changes

<table>
<thead>
<tr>
<th>Problem Encountered</th>
<th>Equation-Graph</th>
<th>Graph-Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Study 2: Conclusions

RQ2.1: What kinds of difficulties do students encounter when solving problems in multiple representations?

- Students had difficulty interpreting physical meaning of mathematical processes.
  - Thus had difficulties solving problems in graphical and functional representations.

- When the context of the problem changed, could not relate the new problem to original problem.
  - Thus had difficulties identifying the principle and physical quantities needed to solve the new problem.
RQ2.2: How do those difficulties change which the sequence in which these representations are presented?

- Verbal -> Graphical -> Equation sequence has fewer overall difficulties.
- Most of the observed difficulties are related to change in representation, rather than change in context.
- Difficulties due to change in representation are fewer in the G-E sequence compared to E-G sequence.

Why is it easier for students to solve graphical before equational?
Different representations offer different salient cues, levels of ambiguity to facilitate and/or overshadow learning of different concepts.

The sequence in which representations are presented may influence learning & problem solving: Optimal sequencing may be important.
Thank You

For further information

N. Sanjay Rebello
srebello@phys.ksu.edu

Dr. Elizabeth Gire
egire@phys.ksu.edu

Adrian Carmichael
camichaelam@gmail.com

Jacquelyn Chini
jackiehaynicz@gmail.com

Dong-Hai Nguyen
dong-hai@phys.ksu.edu