8.4 Advanced RC Filters

- high pass filter including gain and Bode plots
- cascaded low pass filters
- band pass filters
- band rejection filter - the twin-T
- impedance matching problems
- an ideal operational amplifier
- op-amps as impedance buffers
- an active low pass filter
- an active twin-T filter
An RC high pass filter uses the same circuit as the low pass filter, except that the output voltage is measured across the resistor. The gain of the circuit is given by the following,

\[ G_{\text{highpass}}(f) = \frac{e_{\text{out}}(f)}{e_{\text{in}}(f)} = \frac{Z_R(f)}{Z_{R+C}(f)} \]

Substituting for the impedances yields the following expression.

\[ G(f) = \frac{R}{\sqrt{R^2 + (1/2\pi fC)^2}} = \frac{1}{\sqrt{1 + (f_{3\text{dB}}/f)^2}} \]

When \( f = 0 \), \( G = 0 \); when \( f = \infty \), \( G = 1 \); and when \( f = f_{3\text{dB}} \), \( G = 1/\sqrt{2} \).

High pass filters are used primarily to block low frequency interferences. High frequency signals are usually produced by modulation. They seldom occur naturally.
Although the high pass Bode plot is the mirror image of the low pass Bode plot, the two linear graphs of gain versus frequency look quite different. Note that the high pass filter is less distorting than the low pass filter.
Cascaded Low Pass RC Filters

If the -20 dB/decade attenuation of frequencies above $f_{3dB}$ is not sufficient to remove an interference, two stages of filtering can be used. This approach is seldom used to reduce Johnson noise because there are better approaches.

$$G_{cascaded} = G_{low1} \cdot G_{low2}$$

$$A_{cascaded} = A_{low1} + A_{low2}$$

The linear gain versus frequency plot is a Lorentzian function.

$\text{gain vs. frequency}$

$\text{A(dB) vs. log}(f/f_{3dB})$
Band Pass RC Filters

It is possible to combine low pass and high pass filters to create a bandpass filter. These are useful when the signal is isolated in a range of frequencies due to modulation.

\[
G_{band} = G_{lowpass} \cdot G_{highpass}
\]

\[
A_{band} = A_{lowpass} + A_{highpass}
\]

The width of the pass band can be controlled by the \( f_{3dB} \) of the two filters. The linear gain plot clearly shows that the pass band is not symmetric with frequency.
Filters can be constructed that reject a band of frequencies. The most common is the twin-T filter shown at the right.

The rejection frequency is given by,

\[ f_{\text{reject}} = \frac{1}{2\pi RC} \]

Rejection is based on the current at \( f_{\text{reject}} \) being out of phase by 180° in the two legs, or tees, of the filter. As the frequency moves away from \( f_{\text{reject}} \), the phases approach one another and the filter begins to pass current.

![Twin-T band reject filter](image)

![Twin-T band reject Bode plot](image)
The low pass RC circuit was developed without looking at the circuitry prior to and after the filter. The circuitry in front of the filter will have an impedance which will be in series with the filter, and the circuit after the filter will have an impedance which is in parallel with the capacitor.

$R_f$ and $R_L$ form a voltage divider. To maximize the voltage across $R_L$ it needs to be larger than $R_f$. As an example, if $R_L$ is $1 \text{ M}\Omega$, then $R_f$ can be no larger than $10 \text{ k}\Omega$ if the signal is to be measured with 99% accuracy. A second voltage divider is that created by $R_{out}$ and $R_f$. If $R_f$ is $10 \text{ k}\Omega$, then $R_{out}$ should be no larger than $100 \text{ } \Omega$.

Suppose the values of $R_{out}$ and $R_L$ dictate a very small range for $R_f$. Then the filter time constant can only be adjusted by varying $C$.

An impedance buffer is required to eliminate these restrictions.
An Ideal Operational Amplifier

An electronic schematic of an operational amplifier is shown below. A typical gain might be \( G = 10^5 \).

The input impedance, \( R_S \), is around 1 M\( \Omega \). The effective input impedance is given by \( G \cdot R_S = 10^{11} \Omega \). This means that any input circuit can have an impedance as high as 10\(^9\) \( \Omega \) without forming a significant voltage divider.

The output impedance, \( R_{out} \), is around 5 k\( \Omega \). The effective output impedance is \( R_{out}/G = 0.05 \Omega \). This means that any following circuit can have an impedance as low as 5 \( \Omega \) without forming a significant voltage divider.
An operational amplifier, used as a voltage follower, makes an excellent impedance buffer.

In the most straightforward use of operational amplifiers, two voltage followers can be used - one on the input to $R$ and the other on the output across $C$. 
An RC filter can be combined directly with an operational amplifier by putting \( R \) and \( C \) in the feedback loop.

The gain of the circuit is given by the following.

\[
f_{3dB} = \frac{1}{2\pi R_f C}
\]

\[
G = \frac{R_f}{R_{in}} \frac{1}{\sqrt{(f / f_{3dB})^2 + 1}}
\]

- buffers the output impedance
- does not buffer the input impedance
- allows gains larger than unity
Active Twin-T Rejection Filter

The reject band of a twin-T filter can be dramatically improved by combining it with operational amplifiers. The example circuit uses two voltage followers to connect the filter output to the "bottom" of the T.

The narrow response is for $\alpha = 0.97$; the broad response is for $\alpha = 0$, and corresponds to the passive twin-T.