7.10 Single-Sided Exponential

- even/odd decomposition of the single-sided exponential
- transform of the even part
- decomposition of the odd part using the sign() function
- decomposition of the odd part using the derivative theorem
- writing the transform as a phase-shifted cosine
- graphical depiction of the spectrum
Even/Odd Decomposition

\[
\begin{align*}
1/2 & \rightarrow -1/2 \\
1/2 & \rightarrow +
\end{align*}
\]
Transform of Even Exponential

\[ \exp\left(-\frac{|t|}{t^0}\right) \leftrightarrow \frac{1}{2\pi} \frac{f^0/2}{\left(f^0/2\right)^2 + f^2} \]
Decomposition of Odd Exponential

Using the $\text{sign}(t)$ function.
Decomposition of Odd Exponential

Using the derivative theorem.

\[ \frac{1}{\pi f^0} \]

\[ \frac{i}{2\pi \left(\frac{f^0}{2}\right)^2 + f^2} \]
The real and imaginary parts of the spectrum.

\[ \Phi(f) = \frac{1}{2\pi} \frac{f^0/2}{\left( f^0/2 \right)^2 + f^2} - i \frac{f}{2\pi} \frac{1}{\left( f^0/2 \right)^2 + f^2} \]

Amplitude in the complex plane.

\[ A(f) = \sqrt{\text{Re} \left[ \Phi(f) \right]^2 + \text{Im} \left[ \Phi(f) \right]^2} \]

\[ A(f) = \frac{1}{2\pi} \frac{1}{\sqrt{\left( f^0/2 \right)^2 + f^2}} \]

The phase in the complex plane.

\[ \theta(f) = \text{atan} \left( \frac{\text{Im}}{\text{Re}} \right) = \text{atan} \left( -\frac{2f}{f^0} \right) \]
Graphical Depiction

Even and Odd Parts: $f^0 = 2$

Amplitude: $f^0 = 2$

Phase: $f^0 = 2$
Three-Dimensional Graph

\[ \phi_e(f) \]

\[ \phi_o(f') \]