7.9 Comb Basis Set Transform

- comb-to-comb transform
- comments about the amplitudes
- 1 kHz train of impulse functions
- 1 kHz train of Gaussian pulses
- truncated 1 kHz train of impulse functions
- truncated 1 kHz train of Gaussian pulses
A temporal comb function consists of an infinitely long train of evenly spaced impulses. The pulse train is an even function, with an impulse at $t = 0$. The characteristic pulse spacing is $\Delta t$.

A frequency comb function consists of an infinitely long train of evenly spaced impulses. The pulse train is an even function, with an impulse at $f = 0$. The characteristic pulse spacing is $\Delta f = 1/\Delta t$.

The Fourier transform of $\text{comb}(\Delta t)$ is $\text{comb}(\Delta f)$. Note that the amplitude in the spectrum is multiplied by $\Delta f$, not divided by $\Delta f$. 
The amplitude relationship is different than most of the other transform pairs. It can be qualitatively understood by the following argument.

If $\Delta t$ is halved, the total number of temporal impulses per unit time will increase by a factor of two. This increases the power in the signal by a factor of two - as long as the amplitude remains constant.

In the frequency domain the spacing between the comb function will increase by a factor of two, because $\Delta f$ has increased by a factor of two. The increase in frequency spacing of the impulses will decrease the power in the frequency spectrum.

Multiplication of the frequency comb by $\Delta f$ will keep the power constant between the two domains.
A 1kHz Train of Impulses

\[ \Delta t = 0.001 \]

\[ \Delta f = 10^3 \]
Gaussian Pulses: Time Domain

\[ \Delta t = 0.001 \]
\[ t^0 = 0.0001 \]
Gaussian Pulses: Frequency Domain

\[ \Delta f = 1,000 \]
\[ f^0 = 10,000 \]
\[ \Delta t = 0.001 \]
\[ T = 0.020 \]
Truncated Impulses: Frequency

\[ \Delta f = 1,000 \]
\[ F = 50 \]
Truncated Gaussian Pulse Train

\[
\text{rect}(0.02) \cdot \left[ \text{gauss}(10^{-4}) \otimes \text{comb}(10^{-3}) \right] \leftrightarrow \\
0.02\text{sinc}(50) \otimes \left[ 10^{-4} \text{gauss}(10^{4}) \cdot 10^{3} \text{comb}(10^{3}) \right]
\]