

Internal entanglement amplification by external interactions

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We propose a scheme to control the level of entanglement between two fixed spin-1/2 systems by interaction with a third particle. For specific designs, entanglement is shown to be “pumped” into the system from the surroundings even when the spin-spin interaction within the system is small or nonexistent. The effect of the external particle on the system is introduced by including a dynamic spinor in the Hamiltonian. Controlled amplification of the internal entanglement to its maximum value is demonstrated. The possibility of entangling noninteracting spins in a stationary state is also demonstrated by coupling each one of them to a flying qubit in a quantum wire.

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INTRODUCTION

Entanglement is considered to be one of the most profound features of quantum mechanics [1,2]. The quantum state of two entangled states cannot be described as a product of the individual quantum states. Entangled or nonseparable states are fundamental to the fields of quantum information and computation [3,4]. Recently, the desire to understand quantum entanglement has been refueled by the development of quantum computation, which started in the 1980s with the pioneering work of Benioff [5], Bennett and Landauer [6], and Deutsch [7], but gathered momentum and much research interest only after Shor’s revolutionary discovery of a quantum-computer algorithm in 1994 that could efficiently find the prime factors of composite integers [8]. The astronomical power of quantum computation has motivated researchers all over the world to become the first to create a practical quantum computer.

In addition to quantum computation, entanglement has also been at the core of many other active research fields, such as quantum teleportation [9,10], dense coding [11], quantum communication [12], and quantum cryptography [13]. The conceptual puzzles posed by entanglement have become a source of novel ideas that might result in physical applications.

A challenge faced in all the mentioned applications is to prepare and tune the degree of entanglement in the states, which is much more subtle than correlating them classically [14]. The preparation of a highly entangled state is a prerequisite for any successful experimental application.

There is increasing interest recently in generating entanglement between spins in solid state structures, and primarily in quantum dots [15–21]. The common starting point for generating entanglement between two quantum dots (two static qubits) is to allow them to interact with external particles (flying qubits), and to measure the state of the flying qubits after the interaction, so that the wave function of the two static qubits collapses into an entangled state. There also exist schemes to produce entanglement between two flying qubits in the solid state by forcing interaction through a quantum dot [22]. In this paper, we propose a scheme to

generate entanglement between two weakly coupled or uncoupled fixed spins, based on the interaction with a dynamic spinor (a third particle) hopping between neighboring quantum dots.

RESULTS AND DISCUSSION

We start by defining the isolated system of two spins whose positions are restricted to two quantum dots (one spin per dot). Let us assume that the spins are subject to an external magnetic field B along the z direction. The Hamiltonian can be written as [23]

$$\hat{H}^{(2)} = -B\sigma_1^z \otimes I_2 - BI_1 \otimes \sigma_2^z - J\sigma_1^x \otimes \sigma_2^x, \quad (1)$$

where, without loss of generality, the J coupling is restricted to the x direction. In the absence of external perturbations, the two spins are in the ground state of $\hat{H}^{(2)}$. By diagonalizing this Hamiltonian in the common basis set $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$ (also known as $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$), the ground state can be obtained for any value of the coupling constant J . A common measure for the ground-state entanglement is the entanglement of formation [24] (E_c). As shown in Fig. 1, the

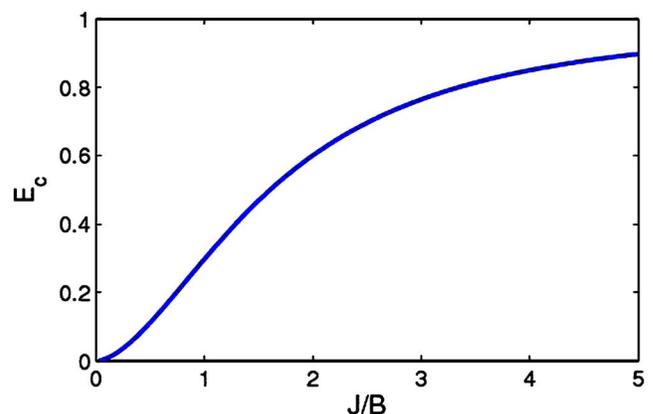


FIG. 1. (Color online) Ground-state entanglement as a function of the relative coupling constant J/B for the Ising-type model.

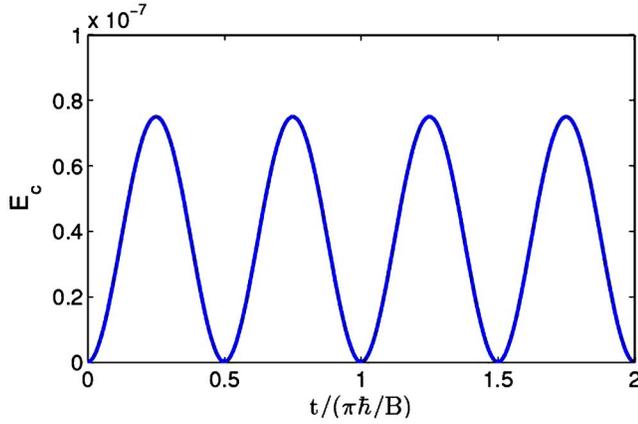


FIG. 2. (Color online) Entanglement as a function of time for a two-spin Ising model with $J/B=0.0001$. The initial state is $\psi(0)=|00\rangle$.

entanglement increases from 0 to 1 as J increases. In the limit of large coupling J (corresponding, e.g., to small distances between the quantum dots), the entanglement in the ground state approaches its maximum value, which corresponds to the fully entangled state $(1/\sqrt{2})(|00\rangle+|11\rangle)$. In the opposite limit, when $J\rightarrow 0$, the Hamiltonian becomes separable and the corresponding ground-state vector approaches the separable state $|00\rangle$, so the ground-state entanglement approaches zero. As $J\rightarrow 0$, it is challenging to design experiments to generate a maximally entangled state, i.e., $E_c\rightarrow 1$.

When the system is prepared in a nonstationary state, entanglement can be developed as a function of time. Let us consider the separable $|00\rangle$ state of the two spins. Even for small J values, the separable state $|00\rangle$ is not a stationary state of the two-spin Hamiltonian. Rather, it is a superposition of two eigenstates of Eq. [1], which are linear combinations of the states $|00\rangle$ and $|11\rangle$. Time-dependent interference between these states produces entanglement, as demonstrated in Fig. 2 for $J/B=0.0001$. The entanglement changes periodically with a period $T\approx\pi\hbar/2B$, corresponding to the energy level spacing between the two stationary states. However, since for small J each one of these states is only weakly entangled, and since only one of them is dominantly populated, the maximal entanglement production is limited to a negligible value ($E_c\ll 1$).

An obvious way to produce a significant level of entanglement between the two spins given small J coupling ($J\ll B$), would be to transfer population from the manifold spanned by $|00\rangle$ and $|11\rangle$ to the nearly degenerate manifold spanned by $|01\rangle$ and $|10\rangle$. This can be done by setting the initial state to $|01\rangle$ or $|10\rangle$, or by setting the initial state to $|00\rangle$ and using an external interaction, e.g., driving one of the two spins locally by a periodic external field according to the time-dependent Hamiltonian [25]

$$\hat{H}=\hat{H}^{(2)}+\hat{V}(t),$$

$$\hat{V}(t)=-\mu\cos(\omega t)(\sigma_1^x\otimes I_2), \quad (2)$$

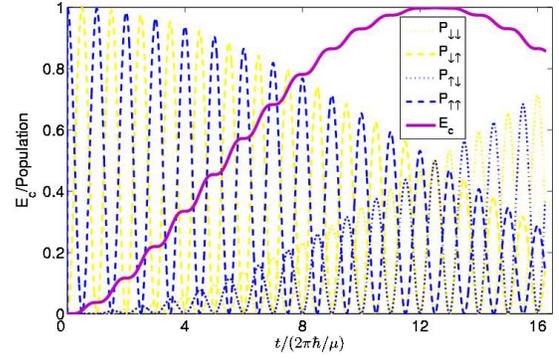


FIG. 3. (Color online) Entanglement and spin-state populations as functions of time, following external periodic driving force of one of the spins. The initial state is $\psi(0)=|00\rangle$. The model parameters are $J=0.0001B$, $\mu=0.005B$, and $\omega\hbar=2B$. E_c is the entanglement of formation. $P_{\uparrow\uparrow}$, $P_{\uparrow\downarrow}$, $P_{\downarrow\downarrow}$, and $P_{\downarrow\uparrow}$ are the populations of the basis functions $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$, respectively. The fast oscillations are Rabi cycles with a time period $T=2\pi\hbar/\mu$.

where ω is the field frequency and μ is the coupling strength. For $J=0$, the entanglement of formation would remain zero at all times, since entanglement cannot be produced by local operations with classical communication. However, for a nonzero value of J , the two spins can be driven into a fully entangled state, as demonstrated in Fig. 3, when the field frequency ω is tuned to match the spin excitation frequency $2B/\hbar$, and for a moderate external driving strength $J<\mu\ll B$. The maximal entanglement is reached at time $\tau=\pi\hbar/2J$. This naive scheme is ineffective in the absence of J coupling, since $\tau_{J\rightarrow 0}\rightarrow\infty$.

In applications for quantum information or quantum communication, it is often desired to entangle two spins that have no direct interaction. How can the two spins be entangled in the absence of any J coupling between them? Below we introduce a scheme to generate an entangled state which does not depend on the presence of J coupling between the spins, and therefore the time scale for entanglement production is not limited by this coupling.

We consider an experiment in which the two noninteracting spins are entangled via a third particle. Specifically, we consider a dynamic spinor (a flying qubit), which interacts with the two spins while in an orbital motion. A schematic representation of the system is given in case 1 of Fig. 4, and the corresponding Hamiltonian reads

$$\hat{H}=\hat{H}^{(s)}+\hat{H}^{(o)}+\hat{H}^{(so)},$$

$$\hat{H}^{(s)}=-B\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\otimes\sigma_3^z\otimes I_2\otimes I_1-B\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\otimes I_3\otimes\sigma_2^z\otimes I_1$$

$$-B\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\otimes I_3\otimes I_2\otimes\sigma_1^z,$$

$$\hat{H}^{(o)}=\begin{bmatrix} 0 & \beta \\ \beta & 0 \end{bmatrix}\otimes I_3\otimes I_2\otimes I_1,$$

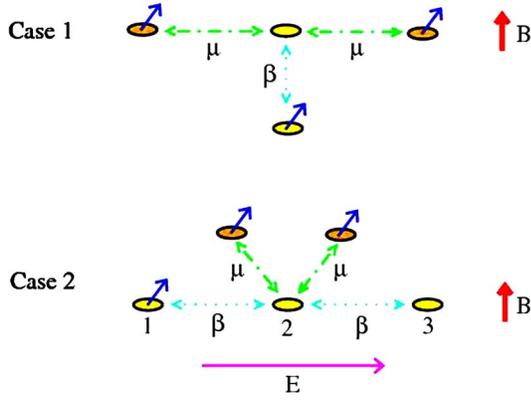


FIG. 4. (Color online) Two cases of entanglement production via interaction with a dynamic spinor. J is the coupling between two fixed spins, μ is the interaction constant between the dynamic spinor and the fixed qubit, and β is the intersite hopping energy of the dynamic spinor between two possible locations. Case 1: The dynamic spinor is coupled directly to both of the fixed spins, in the absence of J coupling between the two fixed spins. Case 2: The one dynamic spin, hopping between three biased sites, is coupled directly to both of the fixed spins, in the absence of J coupling between the two fixed spins,

$$\hat{H}^{(so)} = -\mu \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \otimes \sigma_3^x \otimes (\sigma_2^x \otimes I_1 + I_2 \otimes \sigma_1^x). \quad (3)$$

$\hat{H}^{(s)}$ corresponds to three noninteracting spins, and $\hat{H}^{(o)}$ corresponds to the orbital motion of the flying qubit. The flying qubit populates two energetically equivalent locations (sites), with a hopping matrix element β between them. The spin-orbit-coupling term $\hat{H}^{(so)}$ is designed such that the flying qubit interacts with the two fixed spins from only one of its two possible locations.

In the proposed experiment, all spins are initially in their ground states, and the flying qubit is prepared initially at the noninteracting site. In the absence of spin-orbit coupling ($\mu=0$) this initial state is a superposition of two eigenstates

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|s\rangle \otimes |000\rangle + |a\rangle \otimes |000\rangle) \quad (4)$$

of the separable Hamiltonian $\hat{H}^{(s)} - \hat{H}^{(o)}$. The eigenstates of $\hat{H}^{(o)}$ are denoted $|s\rangle$ and $|a\rangle$, corresponding to a symmetric and an antisymmetric superposition of the two localized states. These two states are separated in energy by 2β . The time evolution in the absence of spin-orbit coupling amounts to oscillations of the spinor between the two sites at the corresponding frequency $2\beta/\hbar$, with no spin excitation and no entanglement production.

Introducing the spin-orbit coupling term $\mu \neq 0$, energy exchange between the orbital motion and the spin degrees of freedom can lead to spin excitation. Our scheme is based on simultaneous excitation of the two static spins in order to entangle them. For this purpose, we set the free orbital motion frequency in resonance with twice the spin excitation energy, $4B$, i.e., $\beta=2B$. This implies that, for $\mu=0$, the state $|a\rangle \otimes |000\rangle$ becomes degenerate with the manifold of two-

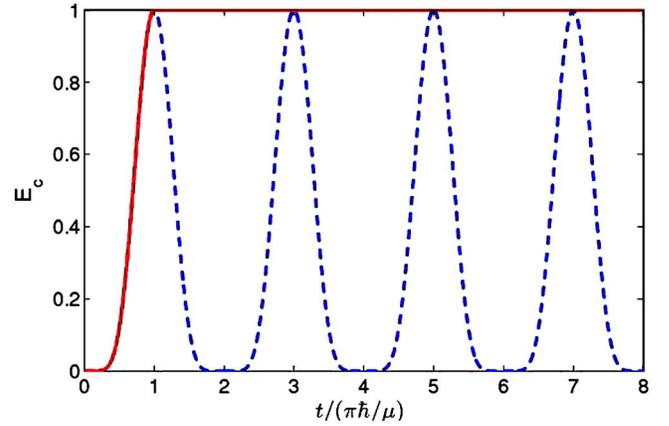


FIG. 5. (Color online) Entanglement as a function of time for case 1 of Fig. 4. The initial state is taken as the ground state of the three uncoupled spins, with the dynamic spinor located at the site that is not directly coupled to the static spins. The dashed line corresponds to a continuous orbital motion, and the solid line is obtained by a sudden interruption of the spin-orbit coupling, maintaining the entanglement of the two spins at its maximal value.

spin excitations: $\{|s\rangle \otimes |110\rangle, |s\rangle \otimes |101\rangle, |s\rangle \otimes |011\rangle\}$. Switching on the spin-orbit coupling, the degeneracy is removed, and the corresponding eigenstates of \hat{H} [Eq. (3)] are superpositions of these states. The initial selected state $|\psi_0\rangle$ [see Eq. (4)] populates these eigenstates along with the $|s\rangle \otimes |000\rangle$ state, and therefore the time evolution of the system mixes the state $|s\rangle \otimes |000\rangle$ with states involving two-spin excitations. In particular, an entangled state of the two static spins (superposition of $|s\rangle \otimes |011\rangle$ and $|s\rangle \otimes |000\rangle$) is obtained on the spin-orbit-coupling time scale.

This is demonstrated in Fig. 5 for the model parameters $\beta=2B$ and $\mu=0.00001B$, the entanglement evolves in time, where maximal entanglement is reached at $t_{\max} = \pi\hbar/\mu$. Under the full Hamiltonian, the entanglement would continue to change periodically in time, as seen in Fig. 5. However, switching off the spin-orbit interaction instantaneously at t_{\max} (setting μ to zero for $t > t_{\max}$) fixes the entanglement at its maximal value. This scheme enables us to produce entanglement and to manipulate its level between the two static spins with no direct coupling between them. The time of maximal entanglement production, t_{\max} , is set by the external coupling strength to the flying qubit. The orbital hopping frequency parameter β and the spin-orbit-coupling strength μ are experimentally controllable, e.g., by the distance between the two static quantum dots and the two sites of the flying qubit. By manipulating these parameters, it would be possible to rapidly turn on or off the entanglement between the two spins.

In the above example, entanglement between the two noninteracting spins was produced dynamically. Below we study an alternative system, where the two static spins can be entangled in the ground state. For this purpose we consider a model of a flying qubit in a quantum wire coupled to two static qubits through Ising-like interactions. The Hamiltonian (illustrated in case 2 of Fig. 4) is a special case of the s - d Hamiltonian, which can be written as (dimensionless units are used for convenience)

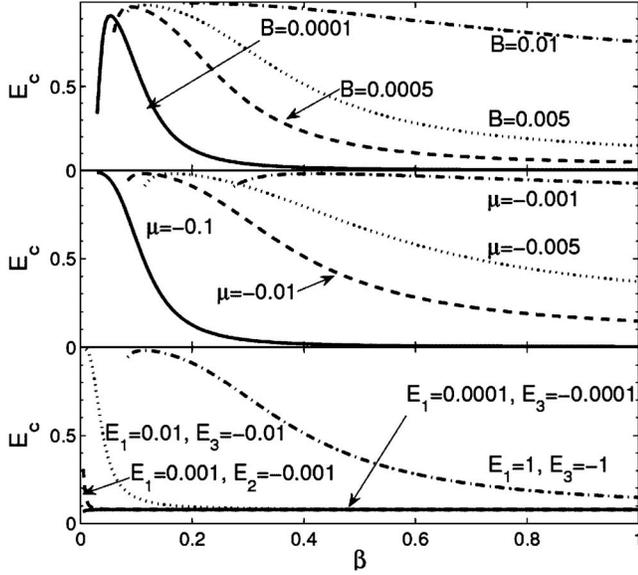


FIG. 6. Entanglement as a function of the hopping magnitude β for case 2 in Fig. 4. Top panel: $E_1=1, E_2=0, E_3=-1, \mu=-0.01$ with $B=0.0001, 0.0005, 0.005$, and 0.01 . Middle panel: $E_1=1, E_2=0, E_3=-1, B=0.001$ with $\mu=-0.1, -0.01, -0.005, -0.001$. Bottom panel: $\mu=-0.01, B=0.001, E_2=0$ with $E_1=0, E_3=0; E_1=1, E_3=-1; E_1=5, E_3=-5; E_1=10, E_3=-10$.

$$H = \sum_{i=1,\sigma}^3 E_i c_{i,\sigma}^\dagger c_{i,\sigma} + B \sum_{i=1}^3 S_i^z - \beta \sum_{i=1,j=1,\sigma}^3 \delta_{i,j\pm 1} c_{i,\sigma}^\dagger c_{j,\sigma} + \mu S_2^x S_A^x + \mu S_2^x S_B^x, \quad (5)$$

using the notation $S_R^{x,y,z} = \frac{1}{2} [c_{R,\uparrow}^\dagger, c_{R,\downarrow}^\dagger] \sigma^{x,y,z} [c_{R,\uparrow}, c_{R,\downarrow}]^T$, $\sigma = \{\uparrow, \downarrow\}$, where $c_{R,\sigma}^\dagger$ is the creation operator for a particle at the R th site. E_i is the electric field applied to the i th site of the quantum wire, B is the external magnetic field applied to the quantum wire (but not to the static dots), β is the nearest-neighbor hopping magnitude of the flying qubit between sites 1, 2, and 3, and μ is the interaction between the flying qubit and the fixed spins, occupying sites A and B . The electric field E_i is introduced in the Hamiltonian to mimic the effect of applying a potential bias along the wire. We study the case where only one electron is present in the entire quantum wire, as would happen when the on-site electron-electron repulsion is high enough to avoid double occupancy of any single dot, and the injected electron current is low enough to permit the presence of a single electron in the wire.

Below we demonstrate that the entanglement between the two fixed spins in this device can be controlled and kept at a stationary value by controlling the device parameters. In the top panel of Fig. 6, we plot the ground-state entanglement as a function of the hopping magnitude β . For a weak magnetic field $|\mu| \gg B$ ($B=0.0001$), the entanglement increases from zero, and reaches a maximum value which corresponds to the state $\varphi_0 \approx |\downarrow_3\rangle \Phi_{AB}$, in which the flying qubit occupies primarily dot 3 at the lowest on-site energy with spin down,

and the two static spins are in an entangled state $\Phi_{AB} \approx (1/\sqrt{2})(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$. However, when β increases farther, the flying qubit becomes delocalized in the quantum wire. The interaction between the flying qubit and the two static qubits through the central site of the quantum wire provides the opportunity to randomize the spin states. Under this condition, the counterpart wave function $\varphi'_0 = |\uparrow_3\rangle \Phi'_{AB}$, where $\Phi'_{AB} = (1/\sqrt{2})(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$, mixes with φ_0 , and the uniform distribution of the static spins between the states Φ_{AB} and Φ'_{AB} reduces their entanglement. Finally, the entanglement vanishes when the three-spin state is approached, $\sim (|\downarrow_3, \uparrow\uparrow\rangle + |\uparrow_3, \uparrow\downarrow\rangle + |\uparrow_3, \downarrow\uparrow\rangle + |\downarrow_3, \downarrow\downarrow\rangle)$, where the spin state for the static spins is totally randomized. Note that the outcome of a measurement of the spin state of the flying qubit, $|\uparrow_3\rangle$ or $|\downarrow_3\rangle$, will collapse the state of the static spins into one of Φ_{AB} and Φ'_{AB} , which are maximally entangled. This mechanism also explains the increase in the entanglement as one tries to polarize the spin state of the flying qubit, which reduces the randomization of the spin state of the flying qubit, by increasing the applied external magnetic field. We also notice that the maximum entanglement points shift to higher values of β as the external magnetic field B increases. In the middle panel of Fig. 6, we show the influence of the μ coupling between the flying qubit and the fixed spins on the ground-state entanglement. With increasing β , the entanglement decreases after reaching a maximum value, where the decay rate is much smaller for smaller μ .

The electrostatic potential along the quantum wire is another important parameter for controlling the entanglement between the fixed spins. In this model, increasing the bias potential will enhance localization of the flying qubit at the minimal energy site, which will suppress the randomization of the flying qubit spin interacting with the static spins. This should increase the entanglement between the static spins, as demonstrated in the bottom panel of Fig. 6. The entanglement is shown to increase when the bias potential increases, and it decays to zero when β is large enough to make the flying qubit delocalized.

CONCLUSIONS

In conclusion, we demonstrated schemes to produce, amplify, and control entanglement between two noninteracting fixed spins, through a dynamic spinor. We showed that maximum entanglement can be achieved by controlling the interaction between the fixed spins and the dynamic spinor, or by controlling the design of the quantum wire that defines the orbital motion of the spinor. Experimental realizations of such schemes might provide a resource to extend entanglement-based applications in quantum information and quantum computation.

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