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Efficient Remote Preparation of Four-Qubit Cluster-Type Entangled States with Multi-Party Over Partially Entangled Channels

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Abstract We present a strategy for realizing multiparty-controlled remote state preparation (MCRSP) for a family of four-qubit cluster-type states by taking a pair of partial entanglements as the quantum channels. In this scenario, the encoded information is transmitted from the sender to a spatially separated receiver with control of the transmission by multiple parties. Predicated on the collaboration of all participants, the desired state can be faithfully restored at the receiver's location with high success probability by application of additional appropriate local operations and necessary classical communication. Moreover, this proposal for MCRSP can be faithfully achieved with unit total success probability when the quantum channels are distilled to maximally entangled ones.

Keywords Remote state preparation · Cluster-type state · Local operation and classical communication

1 Introduction

An important focus in the field of quantum information processing (QIP) has been the secure and faithful transmission of information from one node of quantum network to another non-local node with finite classical and quantum resources. Quantum teleportation (QT), originating from the pioneering work of Bennett [1], is one application of non-local physics which may be used to accomplish such a task. The central idea of QT is to deliver quantum information by means of a pre-established entanglement without physically sending an

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information carrier from the sender to the receiver. Apart from QT, there exists yet another such effective method: Remote State Preparation (RSP) [2–4]. RSP can be interpreted as the transfer of arbitrary known quantum states from a sender (Alice) to a spatially distant receiver (Bob), provided that the two parties share an entangled state and may communicate classically. Although both QT and RSP are able to achieve the task of information transfer [5–7], there are some subtle differences between QT and RSP that can be summarized as follows: (i) *Precondition*. In RSP, the sender of the states is required to have complete knowledge about the prepared state. In contrast, during QT neither the sender nor the receiver need necessarily possesses information associated with the teleported states. (ii) *State existence*. The state to be teleported initially inhabits a physical particle in the context of QT, while this is not a requirement for RSP. That is to say, the sender in RSP is fully aware of the information regarding the desired state, without any particle in such a state existing within his possession. (iii) *Resource trade-off*. Bennett [4] has shown that quantum and classical resources can be traded off in RSP but cannot in QT. In standard teleportation, an unknown quantum state is sent via a quantum channel, involving 1 ebit, and 2 cbits for communication. In contrast, if the teleported state is known to the sender prior to teleportation, the required resources may be reduced to 1 ebit and 1 cbit in RSP at the expense of a lower success probability (half of that from QT). However, Pati [3] has argued that for special ensemble states (e.g., states on either the equator or great polar circle of the Bloch sphere) RSP requires less classical information than teleportation while maintaining the same unitary success probability.

Owing to its importance in QIP, RSP has triggered much attention and been the basis of a number of theoretical investigations [8–24]. Specifically, there have been investigations concerning: low-entanglement RSP [8], optimal RSP [9], oblivious RSP [10, 11], RSP without oblivious conditions [12], generalized RSP [13], faithful RSP [14], joint RSP (JRSP) [15–17], RSP for multi-qubit states [18–22], RSP for qutrit states [23] and continuous variable RSP in phase space [24]. Several RSP proposals, by means of different physical systems, have been experimentally demonstrated [25–31]. As examples, Peng et al. [25] investigated a RSP scheme using NMR, while both Xiang et al. [26] and Peters et al. [27] proposed other two RSP schemes using spontaneous parametric down-conversion. Julio et al. [31] reported the remote preparation of two-qubit hybrid entangled states, including a family of vector-polarization beams; where single-photon states are encoded in the photon spin and orbital angular momentum, and then the desired state is reconstructed by means of spin-orbit state tomography and transverse polarization tomography.

It is well known that cluster state is considered to be a significant resource in the field of quantum information and communication. Remarkably, it can be efficiently applied to accomplish various information processing tasks, including: quantum teleportation [32], quantum dense coding [33, 34], quantum secret sharing [35], and quantum correction [36]. Moreover, it is also a powerful tool for testing nonlocality [37, 38]. Canonically, a cluster-state is written as

$$|\Omega_N\rangle = \frac{1}{2^{N/2}} \bigotimes_{s=1}^{N} (|0\rangle_s Z_{(s+1)} + |1\rangle_s) \tag{1}$$

with the conventional use of Z is a pauli operator and $Z_{N+1} \equiv 1$. It has been shown that one-dimensional N-qubit cluster states are generated in arrays of N qubits mediated with an Ising-type interaction. It may easily be seen that such a state will be reduced into a Bell state for $N = 2$ (or 3); the cluster states are equivalent to Bell states (or Greenberger-Horne-Zeilinger (GHZ) states) respectively under stochastic local operation and classical communication (LOCC). Yet when $N > 3$, the cluster state and the N-qubit GHZ states are not longer interconvertable to each other by LOCC. When $N = 4$, the four-qubit cluster-state is expressed as

$$|\Omega_4\rangle = \frac{1}{2}(|0000\rangle + |0011\rangle + |1100\rangle - |1111\rangle). \tag{2}$$

Recently, many authors have chosen to focus on RSP for cluster-type states by exploring various novel methods [39–43]. In this work, our aim is to examine the implementation of multiparty-controlled remote state preparation (MCRSP) for a family of four-qubit cluster-type entangled states with the aid of general quantum channels.

This paper is organized as follows: in the next section, we present the MCRSP scheme for four-qubit cluster-type entangled states with multi-agent control by the utilization of GHZ-class entanglements as quantum channels. The results show that the desired state can be faithfully reconstructed in distant receiver's laboratory with high success probability. Moreover, the required classical communication cost (CCC) and total success probability (TSP) will be worked out. Finally, the features of our proposed scheme are detailed followed by conclusionary remarks.

2 MCRSP for Four-Qubit Cluster-Type Entangled States via Partial Entanglements

Given there are $(m + n + 2)$ authorized participants: Alice, Bob, Charlie$_1$, $\cdots$, Charlie$_n$, Dick$_1$, $\cdots$, and Dick$_m$. To be explicit, Alice is the sender of the desired state, Bob is the receiver, and Charlie$_s$$(s \in \{1, \cdots, n\})$ and Dick$_t$$(t \in \{1, \cdots, m\})$ are other members of the network. Now, Alice would like to remotely assist Bob in the preparation of a four-qubit cluster-type entangled state, where the transmission is mediated by and may be controlled by the agents within the network. Generally, the form of the four-qubit cluster-type state is given by

$$|P\rangle = \alpha|0000\rangle + \beta e^{i\varphi_0}|0011\rangle + \gamma e^{i\varphi_1}|1100\rangle + \delta e^{i\varphi_2}|1111\rangle, \tag{3}$$

where α, β, γ, δ and each φ_i are real-valued, satisfy the normalized condition $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$, and the boundary condition $\varphi_i \in [0, 2\pi)$. Note that, the state will reduce to one standard four-qubit cluster state as demonstrated in (2), once that $\alpha = \beta = \gamma = \delta = 1/2$, $\varphi_0 = \varphi_1 = 0$ and $\varphi_2 = \pi$. In order to commit MCRSP, Alice, Bob, Charlie$_s$ and Dick$_t$ share previously generated genuine quantum resources – i.e., GHZ-type entanglements – which are given by

$$|\Upsilon^{(1)}\rangle_{A_1A_2B_1B_2C_1\cdots C_n} = \sum_k^{0,1} a_k|k\rangle^{\otimes(n+4)}_{A_1A_2B_1B_2C_1\cdots C_n}, \tag{4}$$

and

$$|\Upsilon^{(2)}\rangle_{A_3A_4B_3B_4D_1\cdots D_m} = \sum_l^{0,1} b_l|l\rangle^{\otimes(m+4)}_{A_3A_4B_3B_4D_1\cdots D_m}, \tag{5}$$

respectively. Initially, qubits A_1, A_2, A_3 and A_4 are distributed to Alice, qubits B_1, B_2, B_3 and B_4 to Bob, C_s to Charlie$_s$$(s \in \{1, \cdots, n\})$ and D_t to Dick$_t$$(t \in \{1, \cdots, m\})$. Without loss of generality, the states' coefficients obey $\sum_k^{0,1}|a_k|^2 = \sum_l^{0,1}|b_l|^2 = 1$, $a_1, b_1 \in \mathbb{R}$, and these bounds $|a_0| \geq |a_1|$ and $|b_0| \geq |b_1|$ are maintained.

The procedure for implementing MCRSP can be divided into the following steps:

Step 1. Firstly, Alice makes a two-qubit projective measurement on her qubit pair (A_1, A_3) under a set of complete orthogonal basis vectors, $\{|\mathcal{L}_{ij}\rangle\}$, composed of the computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, which can be written as

$$(|\mathcal{L}_{00}\rangle, |\mathcal{L}_{01}\rangle, |\mathcal{L}_{10}\rangle, |\mathcal{L}_{11}\rangle)^T = \mathcal{Q}(|00\rangle, |01\rangle, |10\rangle, |11\rangle)^T, \tag{6}$$

where the projection operator $\mathcal{Q}$, in (6), is taken to be

$$\mathcal{Q} = \begin{pmatrix} \alpha & \beta e^{-i\varphi_0} & \gamma e^{-i\varphi_1} & \delta e^{-i\varphi_2} \\ \beta & -\alpha e^{-i\varphi_0} & \delta e^{-i\varphi_1} & -\gamma e^{-i\varphi_2} \\ \gamma & -\delta e^{-i\varphi_0} & -\alpha e^{-i\varphi_1} & \beta e^{-i\varphi_2} \\ \delta & \gamma e^{-i\varphi_0} & -\beta e^{-i\varphi_1} & -\alpha e^{-i\varphi_2} \end{pmatrix}. \tag{7}$$

The total systemic state taken to be the quantum channels can be described by

$$\begin{aligned}
|\Psi_T\rangle &= |\Upsilon^{(1)}\rangle_{A_1A_2B_1B_2C_1\cdots C_n} \otimes |\Upsilon^{(2)}\rangle_{A_3A_4B_3B_4D_1\cdots D_m} \\
&= \sum_{i,j}^{0,1} |\mathcal{L}_{ij}\rangle_{A_1A_3} \otimes |\mathcal{X}_{ij}\rangle_{A_2A_4B_1B_2B_3B_4C_1\cdots C_nD_1\cdots D_m} \\
&= |\mathcal{L}_{00}\rangle_{A_1A_3}(a_0b_0\alpha|0\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m} \\
&\quad +a_0b_1\beta e^{i\varphi_0}|0\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m} \\
&\quad +a_1b_0\gamma e^{i\varphi_1}|1\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m} \\
&\quad +a_1b_1\delta e^{i\varphi_2}|1\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_n}) \\
&\quad +|\mathcal{L}_{01}\rangle_{A_1A_3}(a_0b_0\beta|0\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m} \\
&\quad -a_0b_1\alpha e^{i\varphi_0}|0\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m} \\
&\quad +a_1b_0\delta e^{i\varphi_1}|1\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m} \\
&\quad -a_1b_1\gamma e^{i\varphi_2}|1\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m}) \\
&\quad +|\mathcal{L}_{10}\rangle_{A_1A_3}(a_0b_0\gamma|0\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m} \\
&\quad -a_0b_1\delta e^{i\varphi_0}|0\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m} \\
&\quad -a_1b_0\alpha e^{i\varphi_1}|1\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m} \\
&\quad +a_1b_1\beta e^{i\varphi_2}|1\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m}) \\
&\quad +|\mathcal{L}_{11}\rangle_{A_1A_3}(a_0b_0\delta|0\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m} \\
&\quad +a_0b_1\gamma e^{i\varphi_0}|0\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m} \\
&\quad -a_1b_0\beta e^{i\varphi_1}|1\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m} \\
&\quad -a_1b_1\alpha e^{i\varphi_2}|1\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m}).
\end{aligned} \tag{8}$$

Within the above, the non-normalized state $|\mathcal{X}_{ij}\rangle \equiv {}_{A_1A_3}\langle\mathcal{L}_{ij}|\Psi_T\rangle$ $(i, j = 0, 1)$ is obtained with a probability of $1/\mathcal{N}_{ij}^2$, where $\mathcal{N}_{ij}$ corresponds to the normalized parameter of state $|\mathcal{X}_{ij}\rangle$.

Step 2. According to her own measurement outcome, $|\mathcal{L}_{ij}\rangle$, Alice makes an appropriate unitary operation, $\hat{U}^{(ij)}_{A_2A_4}$, on her remaining qubit pair (A_2, A_4) under the ordering basis, $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. The specific forms of this unitary operator are given by one of the following statements

$$\hat{U}^{(00)}_{A_2A_4} = \mathrm{diag}(1, 1, 1, 1), \tag{9}$$

$$\hat{U}^{(01)}_{A_2A_4} = \mathrm{diag}(e^{i\varphi_0}, -e^{-i\varphi_0}, e^{i(\varphi_2-\varphi_1)}, -e^{i(\varphi_1-\varphi_2)}), \tag{10}$$

$$\hat{U}^{(10)}_{A_2A_4} = \mathrm{diag}(e^{i\varphi_1}, -e^{i(\varphi_2-\varphi_0)}, -e^{-i\varphi_1}, e^{i(\varphi_0-\varphi_2)}), \tag{11}$$

and

$$\hat{U}^{(11)}_{A_2A_4} = \mathrm{diag}(e^{i\varphi_2}, e^{i(\varphi_1-\varphi_0)}, -e^{i(\varphi_0-\varphi_1)}, -e^{-i\varphi_2}). \tag{12}$$

Subsequently, Alice measures her qubits (A_2 and A_4) under the a set of complete orthogonal basis vectors, $\{|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)\}$, and broadcasts her measured outcomes through classical channels. Incidentally, all of the authorized participators should have previously entered into an agreement that cbits (i, j) correspond to the outcome $|\mathcal{L}_{ij}\rangle_{A_1A_3}$, and cbits (p, q) relate to the measuring outcome of qubits A_2 and A_4, respectively. For simplicity, we define

$$p, q = \begin{cases} 0, & \text{if } |+\rangle \text{ is probed} \\ 1, & \text{if } |-\rangle \text{ is probed} \end{cases}. \tag{13}$$

Step 3. The agents then proceed to carry out single-qubit measurements under the set of vector basis, $\{|\pm\rangle\}$, on their qubits. Later, the agents will inform Bob of the results via classical channels. We assume that the cbit x_s ($s \in \{1, \cdots, n\}$) corresponds to the outcome of the agents C_s, and y_t ($t \in \{1, \cdots, m\}$) corresponds to the outcome of the agents D_t. Note that the values of x_s and y_t have been assigned the notations as p and q (see (13)), respectively. We have $g = \Sigma_{s=1}^{n} x_s$, mod $\oplus$ 2 and $h = \Sigma_{t=1}^{m} y_t$, mod $\oplus$ 2. Initially, there are four different situations defined by different values of (g, h); these cases are listed here, as (g, h): I) (0, 0); II) (0, 1); III) (1, 0); and IV) (1, 1).

Step 4. In response to the different measurement outcomes of the sender and agents, Bob operates on his qubits B_1, B_2, B_3 and B_4 with an appropriate unitary transformation $\hat{U}_{B_1B_2B_3B_4}$; the forms of which are given explicitly in Table 1.

Step 5. Finally, Bob introduces one auxiliary qubit B_A with an initial state of $|0\rangle$. And then he makes a triplet collective unitary transformation ($\hat{V}^{(ij)}_{B_1B_3B_A}$) on his qubits B_1, B_3 and B_A under a set of ordering basis vectors, $\{|000\rangle, |010\rangle, |100\rangle, |110\rangle, |001\rangle, |011\rangle, |101\rangle, |111\rangle\}$, which is given by

$$\hat{V}^{(ij)}_{B_1B_3B_A} = \begin{pmatrix} \mathcal{W}_{ij} & \mathcal{U}_{ij} \\ \mathcal{U}_{ij} & -\mathcal{W}_{ij} \end{pmatrix}, \tag{14}$$

where $\mathcal{W}_{ij}$ and $\mathcal{U}_{ij}$ are 4×4 matrices. Explicitly, the forms of $\mathcal{W}_{ij}$ and $\mathcal{U}_{ij}$ are

$$\begin{Bmatrix} \mathcal{W}_{00} \\ \mathcal{W}_{01} \\ \mathcal{W}_{10} \\ \mathcal{W}_{11} \end{Bmatrix} = \begin{Bmatrix} \mathrm{diag}(\frac{a_1b_1}{a_0b_0}, \frac{a_1}{a_0}, \frac{b_1}{b_0}, 1) \\ \mathrm{diag}(\frac{a_1}{a_0}, \frac{a_1b_1}{a_0b_0}, 1, \frac{b_1}{b_0}) \\ \mathrm{diag}(\frac{b_1}{b_0}, 1, \frac{a_1b_1}{a_0b_0}, \frac{a_1}{a_0}) \\ \mathrm{diag}(1, \frac{b_1}{b_0}, \frac{a_1}{a_0}, \frac{a_1b_1}{a_0b_0}) \end{Bmatrix} \tag{15}$$

and

$$\begin{Bmatrix} \mathcal{U}_{00} \\ \mathcal{U}_{01} \\ \mathcal{U}_{10} \\ \mathcal{U}_{11} \end{Bmatrix} = \begin{Bmatrix} \mathrm{diag}(\sqrt{1-(\frac{a_1b_1}{a_0b_0})^2}, \sqrt{1-(\frac{a_1}{a_0})^2}, \sqrt{1-(\frac{b_1}{b_0})^2}, 0) \\ \mathrm{diag}(\sqrt{1-(\frac{a_1}{a_0})^2}, \sqrt{1-(\frac{a_1b_1}{a_0b_0})^2}, 0, \sqrt{1-(\frac{b_1}{b_0})^2}) \\ \mathrm{diag}(\sqrt{1-(\frac{b_1}{b_0})^2}, 0, \sqrt{1-(\frac{a_1b_1}{a_0b_0})^2}, \sqrt{1-(\frac{a_1}{a_0})^2}) \\ \mathrm{diag}(0, \sqrt{1-(\frac{b_1}{b_0})^2}, \sqrt{1-(\frac{a_1}{a_0})^2}, \sqrt{1-(\frac{a_1b_1}{a_0b_0})^2}) \end{Bmatrix}. \tag{16}$$

Bob then measures his auxiliary qubit, B_A, under a set of measuring basis vectors, $\{|0\rangle, |1\rangle\}$. If the state $|1\rangle$ is measured, his remaining qubits will collapse into the trivial state; the MCRSP fails in this situation. Otherwise, $|0\rangle$ is probed and the qubits' state will transform into the desired state; that is, our MCRSP is successful in this case.

Based on the above five-step protocol, it has been shown that the MCRSP for a family of cluster-type states can be faithfully performed with a predictable success probability. The steps can be decomposed into a schematic diagram shown in Fig. 1. As a summary, we list Bob's required local single-qubit transformations corresponding to various measurement outcomes by both the sender and the agents in Table 1. By the analysis above, one can see that the prepared state can be faithfully reconstructed with a specified success probability.

Above, we have shown that MCRSP for a four-qubit cluster-state can be faithfully performed by employing general entangled states as quantum channels. For clarity, here we will take the case of $(i, j, p, q, g, h) = (1, 0, 0, 1, 1, 0)$ as an example. That is, the state $\{|\mathcal{L}_{10}\rangle\}$ is detected on qubits (A_1, A_3) by Alice at the beginning. Thus, the remaining qubits will be converted into

$$\begin{aligned}|\psi^1\rangle = \mathcal{N}_{10}(&a_0b_0\gamma|0\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m}\\ &-a_0b_1\delta e^{i\varphi_0}|0\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m}\\ &-a_1b_0\alpha e^{i\varphi_1}|1\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m}\\ &+a_1b_1\beta e^{i\varphi_2}|1\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m}),\end{aligned} \tag{17}$$

where $\mathcal{N}_{10} = \frac{1}{\sqrt{|a_0b_0\gamma|^2+|a_0b_1\delta|^2+|a_1b_0\alpha|^2+|a_1b_1\beta|^2}}$ is the normalized coefficient.

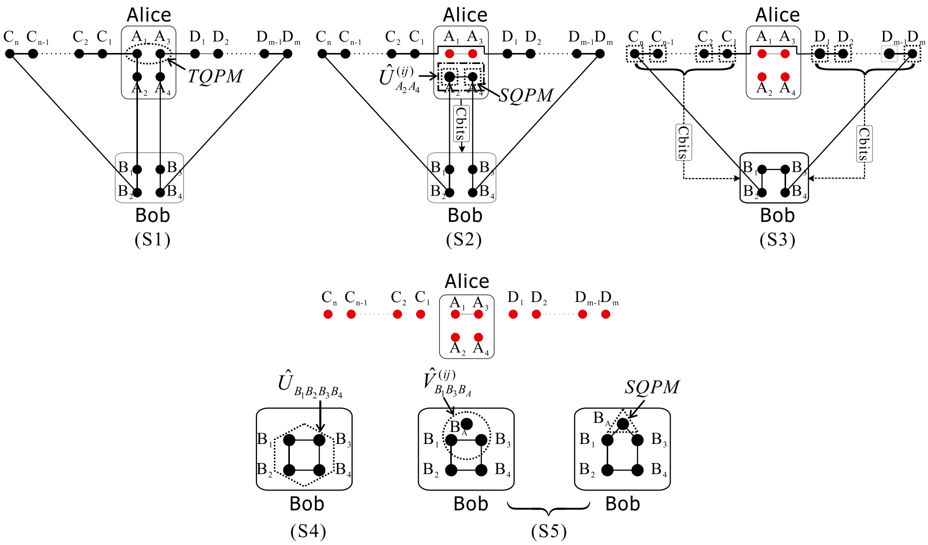

Fig. 1 Schematic diagram for our MCRSP implementation. Seen above, the procedure is explicitly decomposed into diagrams (S1) through (S5). The *dotted ellipse* represents a two-qubit projective measurement (TQPM) under the $\{|\mathcal{L}_{ij}\rangle\}$ basis; the *dotted square* represents a single-qubit projective measurement (SQPM) under the $\{|\pm\rangle\}$ basis; the *dotted-line* rectangle represents the operations of a bipartite collective unitary transformation $\hat{U}^{(ij)}_{A_2A_4}$; the *dotted hexagon* represents the single-qubit unitary transformation $\hat{U}_{B_1B_2B_3B_4}$ on Bob's qubits; the *dotted circle* signifies a triplet collective unitary operation $\hat{V}^{(ij)}_{B_1B_3B_A}$; the *dotted triangle* represents SQPM under the $\{|0\rangle, |1\rangle\}$ basis; the *red solid dot* represents a particle which has been measured in a Hilbert Space, and Cbits represents classical information communication. Note that there are no operations on Alice's, Charlie's and Dick's qubits after Step 3

Table 1 $ijpqgh$ denotes the corresponding measurement outcomes from the authorized participants; $\hat{U}_{B_1B_2B_3B_4}$ Denotes the unitary operation that Bob must perform on qubits B_1, B_2, B_3 and B_4

$ijpqgh$	$\hat{U}_{B_1B_2B_3B_4}$	$ijpqgh$	$\hat{U}_{B_1B_2B_3B_4}$	$ijpqgh$	$\hat{U}_{B_1B_2B_3B_4}$	$ijpqgh$	$\hat{U}_{B_1B_2B_3B_4}$
000000	$I_{B_1}I_{B_2}I_{B_3}I_{B_4}$	010000	$I_{B_1}I_{B_2}X_{B_3}X_{B_4}$	100000	$X_{B_1}X_{B_2}I_{B_3}I_{B_4}$	110000	$X_{B_1}X_{B_2}X_{B_3}X_{B_4}$
000001	$I_{B_1}I_{B_2}Z_{B_3}I_{B_4}$	010001	$I_{B_1}I_{B_2}X_{B_3}Z_{B_3}X_{B_4}$	100001	$X_{B_1}X_{B_2}Z_{B_3}I_{B_4}$	110001	$X_{B_1}X_{B_2}X_{B_3}Z_{B_3}X_{B_4}$
000010	$Z_{B_1}I_{B_2}I_{B_3}I_{B_4}$	010010	$Z_{B_1}I_{B_2}X_{B_3}X_{B_4}$	100010	$X_{B_1}Z_{B_1}X_{B_2}I_{B_3}I_{B_4}$	110010	$X_{B_1}Z_{B_1}X_{B_2}X_{B_3}X_{B_4}$
000011	$Z_{B_1}I_{B_2}Z_{B_3}I_{B_4}$	010011	$Z_{B_1}I_{B_2}X_{B_3}Z_{B_3}X_{B_4}$	100011	$X_{B_1}Z_{B_1}X_{B_2}Z_{B_3}I_{B_4}$	110011	$X_{B_1}Z_{B_1}X_{B_2}X_{B_3}Z_{B_3}X_{B_4}$
000100	$I_{B_1}I_{B_2}Z_{B_3}I_{B_4}$	010100	$I_{B_1}I_{B_2}X_{B_3}Z_{B_3}X_{B_4}$	100100	$X_{B_1}X_{B_2}Z_{B_3}I_{B_4}$	110100	$X_{B_1}X_{B_2}X_{B_3}Z_{B_3}X_{B_4}$
000101	$I_{B_1}I_{B_2}I_{B_3}I_{B_4}$	010101	$I_{B_1}I_{B_2}X_{B_3}X_{B_4}$	100101	$X_{B_1}X_{B_2}I_{B_3}I_{B_4}$	110101	$X_{B_1}X_{B_2}X_{B_3}X_{B_4}$
000110	$Z_{B_1}I_{B_2}Z_{B_3}I_{B_4}$	010110	$Z_{B_1}I_{B_2}X_{B_3}Z_{B_3}X_{B_4}$	100110	$X_{B_1}Z_{B_1}X_{B_2}Z_{B_3}I_{B_4}$	110110	$X_{B_1}Z_{B_1}X_{B_2}X_{B_3}Z_{B_3}X_{B_4}$
000111	$I_{B_1}I_{B_2}Z_{B_3}I_{B_4}$	010111	$Z_{B_1}I_{B_2}X_{B_3}X_{B_4}$	100111	$X_{B_1}Z_{B_1}X_{B_2}I_{B_3}I_{B_4}$	110111	$X_{B_1}Z_{B_1}X_{B_2}X_{B_3}X_{B_4}$
001000	$Z_{B_1}I_{B_2}I_{B_3}I_{B_4}$	011000	$I_{B_1}I_{B_2}X_{B_3}Z_{B_3}X_{B_4}$	101000	$X_{B_1}Z_{B_1}X_{B_2}I_{B_3}I_{B_4}$	111000	$X_{B_1}Z_{B_1}X_{B_2}X_{B_3}X_{B_4}$
001001	$Z_{B_1}I_{B_2}Z_{B_3}I_{B_4}$	011001	$Z_{B_1}I_{B_2}X_{B_3}Z_{B_3}X_{B_4}$	101001	$X_{B_1}Z_{B_1}X_{B_2}Z_{B_3}I_{B_4}$	111001	$X_{B_1}Z_{B_1}X_{B_2}X_{B_3}Z_{B_3}X_{B_4}$
001010	$I_{B_1}I_{B_2}Z_{B_3}I_{B_4}$	011010	$I_{B_1}I_{B_2}X_{B_3}X_{B_4}$	101010	$X_{B_1}X_{B_2}I_{B_3}I_{B_4}$	111010	$X_{B_1}X_{B_2}X_{B_3}X_{B_4}$
001011	$I_{B_1}I_{B_2}I_{B_3}I_{B_4}$	011011	$I_{B_1}I_{B_2}X_{B_3}Z_{B_3}X_{B_4}$	101011	$X_{B_1}X_{B_2}Z_{B_3}I_{B_4}$	111011	$X_{B_1}X_{B_2}X_{B_3}Z_{B_3}X_{B_4}$
001100	$Z_{B_1}I_{B_2}Z_{B_3}I_{B_4}$	011100	$Z_{B_1}I_{B_2}X_{B_3}Z_{B_3}X_{B_4}$	101100	$X_{B_1}Z_{B_1}X_{B_2}Z_{B_3}I_{B_4}$	111100	$X_{B_1}Z_{B_1}X_{B_2}X_{B_3}Z_{B_3}X_{B_4}$
001101	$Z_{B_1}I_{B_2}I_{B_3}I_{B_4}$	011101	$Z_{B_1}I_{B_2}X_{B_3}X_{B_4}$	101101	$X_{B_1}Z_{B_1}X_{B_2}I_{B_3}I_{B_4}$	111101	$X_{B_1}Z_{B_1}X_{B_2}X_{B_3}X_{B_4}$
001110	$I_{B_1}I_{B_2}I_{B_4}$	011110	$I_{B_1}I_{B_2}X_{B_3}Z_{B_3}X_{B_4}$	101110	$X_{B_1}X_{B_2}Z_{B_3}I_{B_4}$	111110	$X_{B_1}X_{B_2}X_{B_3}Z_{B_3}X_{B_4}$
001111	$I_{B_1}I_{B_2}I_{B_3}I_{B_4}$	011111	$I_{B_1}I_{B_2}X_{B_3}X_{B_4}$	101111	$X_{B_1}X_{B_2}I_{B_3}I_{B_4}$	111111	$X_{B_1}X_{B_2}X_{B_3}X_{B_4}$

Later, Alice makes the operation $\hat{U}^{(10)}_{A_2A_4}$ on her remaining qubits A_2 and A_4. As a consequence, the above state will evolve into

$$\begin{aligned}|\psi^2\rangle\ =\ &\mathcal{N}_{10}(a_0b_0\gamma e^{i\varphi_1}|0\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m}\\&+a_0b_1\delta e^{i\varphi_2}|0\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m}\\&+a_1b_0\alpha|1\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m}\\&+a_1b_1\beta e^{i\varphi_0}|1\rangle^{\otimes(n+3)}_{A_2B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+3)}_{A_4B_3B_4D_1\cdots D_m}).\end{aligned}\tag{18}$$

Incidentally, the state given in (18) can be rewritten as

$$\begin{aligned}|\psi^3\rangle\ =\ &\frac{\mathcal{N}_{10}}{2}[|++\rangle_{A_2A_4}(a_0b_0\gamma e^{i\varphi_1}|0\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m}\\&+a_0b_1\delta e^{i\varphi_2}|0\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m}\\&+a_1b_0\alpha|1\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m}+a_1b_1\beta e^{i\varphi_0}|1\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m})\\&+|+-\rangle_{A_2A_4}(a_0b_0\gamma e^{i\varphi_1}|0\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m}\\&-a_0b_1\delta e^{i\varphi_2}|0\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m}\\&+a_1b_0\alpha|1\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m}-a_1b_1\beta e^{i\varphi_0}|1\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m})\\&+|-+\rangle_{A_2A_4}(a_0b_0\gamma e^{i\varphi_1}|0\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m}\\&+a_0b_1\delta e^{i\varphi_2}|0\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m}\\&-a_1b_0\alpha|1\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m}-a_1b_1\beta e^{i\varphi_0}|1\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m})\\&+|--\rangle_{A_2A_4}(a_0b_0\gamma e^{i\varphi_1}|0\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m}\\&-a_0b_1\delta e^{i\varphi_2}|0\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m}\\&-a_1b_0\alpha|1\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m}\\&+a_1b_1\beta e^{i\varphi_0}|1\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m})].\end{aligned}\tag{19}$$

Accordingly, Alice measures her qubits A_2 and A_4 under the basis vectors $\{|\pm\rangle\}$. Letting the outcome be $|+\rangle_{A_2}|-\rangle_{A_4}$, Alice broadcasts this outcome to Bob and agents via the classical message '01'. The subsystem state will then be

$$\begin{aligned}|\psi^4\rangle=&\mathcal{N}_{10}(a_0b_0\gamma e^{i\varphi_1}|0\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m}-a_0b_1\delta e^{i\varphi_2}|0\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m}\\&+a_1b_0\alpha|1\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|0\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m}-a_1b_1\beta e^{i\varphi_0}|1\rangle^{\otimes(n+2)}_{B_1B_2C_1\cdots C_n}|1\rangle^{\otimes(m+2)}_{B_3B_4D_1\cdots D_m}).\end{aligned}\tag{20}$$

Meanwhile, the agents proceed to carry out single-qubit projective measurements under the same vector basis $\{|\pm\rangle\}$ on their own qubits respectively, and then informs Bob of their measurement results via classical channels. Supposing the measuring outcome $(p, q) = (1, 0)$, Bob realizes the state of his qubits B_1, B_2, B_3 and B_4 will collapse into

$$\begin{aligned}|\psi^5\rangle\ =\ &\mathcal{N}_{10}(a_0b_0\gamma e^{i\varphi_1}|0000\rangle-a_0b_1\delta e^{i\varphi_2}|0011\rangle-a_1b_0\alpha|1100\rangle\\&+a_1b_1\beta e^{i\varphi_0}|1111\rangle)_{B_1B_2B_3B_4}.\end{aligned}\tag{21}$$

Subsequently, Bob performs $X_{B_1}Z_{B_1}X_{B_2}Z_{B_3}I_{B_4}$ on his qubits B_1, B_2, B_3 and B_4, respectively, where X and Z are pauli operators and I is an identity operator. His systematic state will collapse into

$$|\psi^6\rangle = \mathcal{N}_{10}(a_1 b_0 \alpha|0000\rangle + a_1 b_1 \beta e^{i\varphi_0}|0011\rangle + a_0 b_0 \gamma e^{i\varphi_1}|1100\rangle + a_0 b_1 \delta e^{i\varphi_2}|1111\rangle)_{B_1B_2B_3B_4}. \tag{22}$$

Bob then introduces the auxiliary qubit, B_A, with an initial state of $|0\rangle$. He will now implement a local triplet collective unitary transformation, $\hat{V}^{(10)}_{B_1B_3B_A}$, on qubits B_1, B_3 and B_A. Thus Bob's system will become

$$\begin{aligned}|\psi^7\rangle = \mathcal{N}_{10}[&a_1 b_1(\alpha|0000\rangle + \beta e^{i\varphi_0}|0011\rangle + \gamma e^{i\varphi_1}|1100\rangle + \delta e^{i\varphi_2}|1111\rangle)_{B_1B_2B_3B_4} \otimes |0\rangle_{B_A}\\ &+(a_1 b_0 \alpha\sqrt{1-(\frac{b_1}{b_0})^2}|0000\rangle + a_0 b_0 \gamma e^{i\varphi_1}\sqrt{1-(\frac{a_1 b_1}{a_0 b_0})^2}|1100\rangle\\ &+a_0 b_1 \delta e^{i\varphi_2}\sqrt{1-(\frac{a_1}{a_0})^2}|1111\rangle)_{B_1B_2B_3B_4} \otimes |1\rangle_{B_A}].\end{aligned} \tag{23}$$

Finally, Bob makes a single-qubit projective measurement on qubit B_A under basis vectors $\{|0\rangle, |1\rangle\}$. If $|1\rangle_{B_A}$ is measured, his remaining qubits will collapse into the trivial state and the RSP fails. If $|0\rangle_{B_A}$ is measured, the remaining qubits will transform into the state: $(\alpha|0000\rangle + \beta e^{i\varphi_0}|0011\rangle + \gamma e^{i\varphi_1}|1100\rangle + \delta e^{i\varphi_2}|1111\rangle)_{B_1B_2B_3B_4} \equiv |P\rangle$.

Of course, Alice's outcome may be one of the remaining three states: $|\mathcal{L}_{00}\rangle$, $|\mathcal{L}_{01}\rangle$ and $|\mathcal{L}_{11}\rangle$. Therefore, the desired state can be faithfully recovered at Bob's location with certainty by similar analysis methods as those above. And the whole process of our MCRSP scheme can be described by

$$\begin{aligned}\rho_{out} = &\mathrm{Tr}_{B_A}(\hat{M}_{B_A}(\hat{V}_{B_1B_3B_A}(\hat{U}_{B_1B_2B_3B_4}\mathrm{Tr}_{C_1\cdots C_nD_1\cdots D_m}(\hat{M}_{C_1\cdots C_nD_1\cdots D_m}\mathrm{Tr}_{A_2A_4}(\hat{M}_{A_2A_4}\hat{U}_{A_2A_4}\mathrm{Tr}_{A_1A_3}\\ &(\hat{M}_{A_1A_3}\rho_{in}M^{\dagger}_{A_1A_3})U^{\dagger M^{\dagger}_{A_2A_4}}_{A_2A_4})\hat{M}^{\dagger}_{C_1\cdots C_nD_1\cdots D_m})\hat{U}^{\dagger}_{B_1B_2B_3B_4} \otimes |0\rangle_{B_A}\langle 0|)\hat{V}^{\dagger}_{B_1B_3B_A})\hat{M}^{\dagger}_{B_A}),\end{aligned} \tag{24}$$

where Tr denotes partial trace, $\rho_{in} = |\Upsilon^{(1)}\rangle\langle\Upsilon^{(1)}|\Upsilon^{(2)}\rangle\langle\Upsilon^{(2)}|$, and $\hat{M}_i$ denotes the projection operators of measurements on qubits i under corresponding measuring basis vectors.

Now, let us proceed with the calculation of TSP and CCC for the current scheme. Derived from (8), Alice's measurement outcome, $|\mathcal{L}_{ij}\rangle$, has an occurrence probability of

$$P_{\mathcal{L}_{ij}} = \frac{1}{\mathcal{N}_{ij}^2}. \tag{25}$$

Furthermore, it should be noted that the probability of capturing the state $|0\rangle_{B_A}$ is given by

$$P_{|0\rangle_{B_A}|\mathcal{L}_{ij}} = (\mathcal{N}_{ij} a_1 b_1)^2. \tag{26}$$

Thus, the TSP of MCRSP sums to be

$$P_{Tot} = \sum_i^{0,1}\sum_j^{0,1} P_{\mathcal{L}_{ij}} \times P_{|0\rangle_{B_A}|\mathcal{L}_{ij}} = 4(a_1 b_1)^2. \tag{27}$$

Moreover, one should note that the required CCC should be $(m + n + 4)$ cbits totally; where the 4 required CCC's are from passing of measurement outcomes from Alice to Bob during Step 2 of the procedure.

Herein, we presented a novel proposed method for MCRSP involving a family of four-qubit cluster-type entangled states. We have proved that our scheme can be realized faithfully with a TSP of $4(a_1 b_1)^2$ and a CCC of $(m + n + 4)$ via the control of multiple

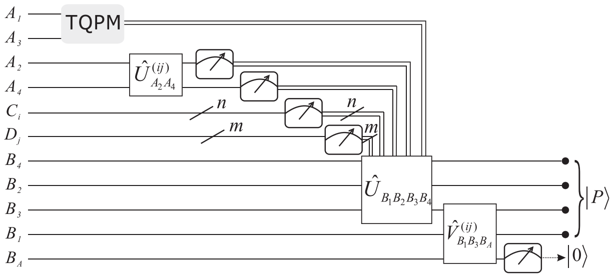

Fig. 2 The quantum circuit diagram for implementing our MCRSP scheme. TQPM denotes a two-qubit projective measurement under a set of complete orthogonal basis vectors, $\{|\mathcal{L}_{ij}\rangle\}$; $\hat{U}^{(ij)}_{A_2A_4}$ denotes Alice's appropriate bipartite collective unitary transformation on the qubit pair (A_2, A_4); $\hat{U}_{B_1B_2B_3B_4}$ denotes Bob's single-qubit unitary transformation on qubits B_1, B_2, B_3 and B_4; and $\hat{V}^{(ij)}_{A_1A_3B_A}$ denotes Bob's triplet collective unitary transformation on qubits B_1, B_3 and B_A

agents within a quantum network. For clarity, the quantum circuit diagram for our MCRSP protocol is provided as Fig. 2.

3 Discussions

We have found several remarkable features with respect to the scheme presented above; these features are summarized as follows: (1) To the best of our knowledge, this is the first time one has exploited such a scenario concerning MCRSP for four-qubit, cluster-type entangled states under the control of a $(m+n)-$party. Conveyance of information regarding the prepared state takes place only between the sender and the receiver, i.e., $1 \to 1$ threshold communication. Moreover, the agents are capable of supervising and switching the procedure during the relay of information communication. This type of secure multi-node information communication ought to be considerably important in prospective quantum networks. (2) Generally, our MCRSP can be faithfully performed with a TSP of $4(a_1b_1)^2$. Moreover, when the state $|a_1| = |b_1| = 1/\sqrt{2}$ is chosen initially, the channels become maximally entangled and the TSP can reach unity as shown in Fig. 3. Consequently, that indicates our scheme becomes deterministic at this limit. It should also be noted that the parameters a_1 and b_1 relate to the Shannon entropies of the employed quantum channels

$$H(f) = -|f|^2 \log\left(|f|^2\right) - (1-|f|^2)\log\left(1-|f|^2\right), \tag{28}$$

within the above, $f \in \{a_1, b_1\}$ and $a_1, b_1 \in [-\sqrt{2}/2, \sqrt{2}/2]$. The entropy will vary with respect to coefficients specific to various choices in quantum channels, depicted in Fig. 4. The entropy in essence reflects inherent and fundamental properties (i.e., entanglements) associated with the specific choice of quantum channels. (3) This scheme enables one to complete RSP via a multi-agent control protocol. Incidentally, all of the agents are capable of switching the preparation procedures. The desired state can be recovered at Bob's site conditional upon the total collaboration of the network members. Any member of the party cannot recover the desired state by themselves. The receiver is not capable of recovering the desired state without the added classical information from all the controllers; controllers are incapable of recovering the desired state even with all of the classical information. In this sense, the security of information is to a large extent guaranteed. (4) Within our scheme,

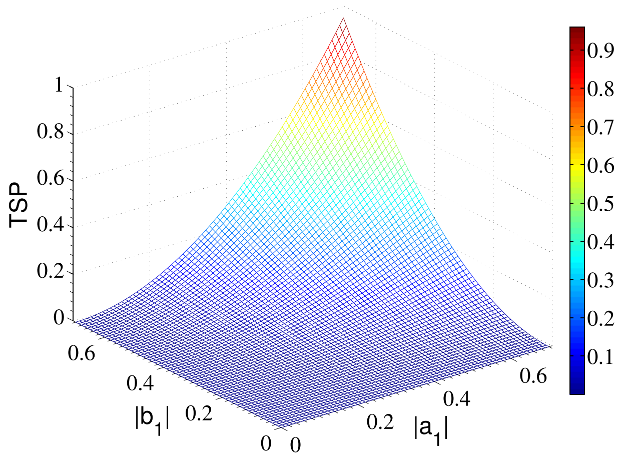

Fig. 3 TSP versus the different ensembles of entanglements employed as quantum channels. TSP represents the total success probability of the scheme

there exists $(m + n)$ controllers capable of manipulating or switching the preparation procedure. If both m and n are chosen to be 0, there are no authorized controllers during the process of the preparation; it has been found that our scheme is smoothly reduces to one resembling RSP for four-qubit cluster-type states with a TSP of $4(a_1b_1)^2$. In this case, measurements made by the controllers and communication between the controllers and the receiver are unnecessary.

Let us now compare our reduced scheme with previous schemes [39–43]. The following discussion is done with respect to RSP and JRSP schemes in view of their resource consumption and their quantum operation complexity as shown in Table 2. From Table 2, one can directly note that the TSP of our scheme is capable of unity and the Intrinsic Efficiency

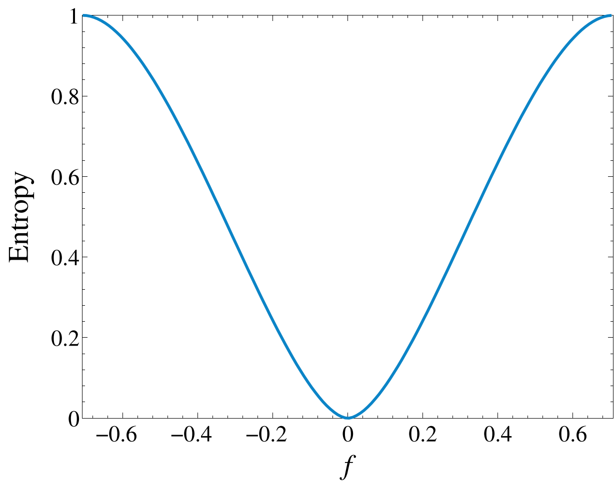

Fig. 4 The entropic diagram under the variation of coefficients a_1 and b_1 associated with the quantum channels. f denotes $\{a_1, b_1\}$; and the vertical coordinate represents the information entropy of the employed channels

Table 2 Comparison between our scheme and the previous ones in the case of maximally entangled channels

Schemes	Required qubits	Quantum operations	CCC	TSP	η
Ref. [39]	six 2-qubit ETs	two 4-qubit PMs	8	$\frac{1}{16}$	1.25 %
	two 6-qubit ETs	two 4-qubit PMs	8	$\frac{1}{16}$	1.25 %
Ref. [40]	two 3-qubit ETs & two ASQ	two 2-qubit PMs & 2 CNOTs	4	$\frac{1}{4}$	8.33 %
Ref. [41]	two 2-qubit ETs & four ASQ	two 4-qubit PMs & 4 CNOTs	4	$\frac{1}{4}$	8.33 %
Ref. [42]	six 2-qubit ETs	two 4-qubit PMs	8	1	20.00 %
Ref. [43]	two 2-qubit & one 3-qubit ET	one 3-qubit PM	3	$\frac{1}{4}$	10.00 %
Current scheme	two 4-qubit ETs	one 2-qbuit PMs & two SQPMs	4	1	33.33 %

ET represents entanglement; SQ represents single-qubit; ASQ represents auxiliary single-qubit; CNOT represents controlled-not gates; PM represents projective measurement; SQPM represents single-qubit projective measurement and TSP represents total success probability

(η) achieves 33.33 %, which is much greater than those values of previous schemes [39–43]. Due to a characteristic high-efficiency and a high-TSP in the present scheme, it is both highly efficient and optimal in comparison to other existing methods. Incidentally, the efficiency of a scheme regarding quantum information processing is related to the amount of qubits in the prepared state and the employed entanglement channels, as well as classical communication (as defined by [44]). Here we similarly define the intrinsic efficiency of our scheme as

$$\eta = \frac{N_s}{N_q + N_c} \times \text{TSP}, \tag{29}$$

where N_s represents the number of qubits of the prepared states, N_q represents the amount of quantum resource consumption, and N_c is the amount of CCC during the quantum computation. Zhan et al. [42] can be realized with a TSP of 100 %, similar to our work at the maximally entangled limit; however, there are several crucial differences between our method and those of ref. [42]. Explicitly: (i) *Resource consumption*. In ref. [42], 12 qubits and 8 cbits are required in the course of RSP for four-qubit cluster-type states, while our scheme requires 8 qubits and 4 cbits to implement RSP for such states. This implies that our scheme is more economic with respect to both quantum and classical resource expenditure. (ii) *Operation complexity*. Two four-qubit projective measurements in ref. [42] are required for their procedure, while two-qubit projection measurements are sufficient for our scheme. Averagely, the experimental realization of a four-qubit projective measurement is much more difficult than that for a two-qubit measurement. Thus, in principal our scheme is easier to experimentally realize than the previous method.

4 Conclusion

Herein we have derived a novel strategy for implementing MCRSP scheme for a family of four-qubit cluster-type entangled states by taking advantage of robust GHZ-class states as quantum channels. With the aid of suitable LOCC, our scheme can be realized with high success probability. Our scheme has several nontrivial features, including: high success probability, security and reducibility. The TSP of MCRSP can reach unity when the quantum channels are distilled to maximally entangled ones; that is, our scheme can be performed both deterministically and efficiently at this limit. We believe that the current scheme might be beneficial to gain better understanding of long-range controllable quantum communication in multi-node networks.

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