Dimensional scaling treatment of stability of simple diatomic molecules induced by superintense, high-frequency laser fields

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We present results obtained using dimensional scaling with high-frequency Floquet theory to evaluate the stability of gas phase simple diatomic molecules in superintense laser fields. The large-D limit provides a simple model that captures the main physics of the problem, which imposes electron localization along the polarization direction of the laser field. This localization markedly reduces the ionization probability and can enhance chemical bonding when the laser strength becomes sufficiently strong. We find that energy and structure calculations at the large-dimensional limit (D → ∞) for stabilities of H2+, H2, and He2 in superintense laser fields are much simpler than at D=3, yet gave similar results for the field strengths needed to bind an “extra” one or two electrons to H and He atoms.

In this paper, we apply the D-scaling method with HFFT to examine the stability in superintense laser fields of simple diatomics, H2+, H2, H2, and He2, with respect to both ionization and bond dissociation. Our results, which require little computation, compare well with those obtained at D=3 from more arduous conventional calculations for H2+, H2, and He2. This encourages exploiting D-scaling with HFFT to treat larger molecules and to assess experimental prospects.

II. LASER-MOLECULE INTERACTION

We consider a high-frequency monochromatic electric field vector of the form

$$E(t) = E_0 (e_1 \cos \omega t + e_2 \tan \delta \sin \omega t), \quad (1)$$

with $E_0$ the amplitude, $\omega$ the frequency, $e_j (j=1,2)$ unit vectors orthogonal to each other and to the propagation direction of the light, $\delta=0$ corresponds to linear polarization, and $\delta = \pm \pi/4$ to circular polarization.21 Except where indicated otherwise, we employ atomic units ($\hbar=e=m_e=1$); thus, distance is in bohrs (0.0529 nm), energy in hartrees (27.2 eV); 1 a.u. of field intensity $I=E_0^2$ is 3.51×10^16 W/cm^2; of frequency $\omega$ is 4.13×10^16 s^{-1}, equivalent to a wavelength of 54.9 nm. For a small molecule, miniscule compared with the wavelength of the light, the dipole approximation is well satisfied. Each electron within the molecule is then subject to the same laser field and undergoes quiver oscillations $\alpha(t)$ along a trajectory given by

$$\alpha(t) = \alpha_0 (e_1 \cos \omega t + e_2 \tan \delta \sin \omega t), \quad (2)$$

where the quiver amplitude is $\alpha_0=E_0/\omega^2$. In the Kramers–Henneberger (KH) reference frame, translated by $\alpha(t)$ with respect to the laboratory frame, the electrons all remain...
fixed; instead, the nuclei quiver along the $\alpha(t)$ trajectory. The Coulombic attraction between any electron and a nucleus with charge $Z$ then takes the form $-Z/r$ $\alpha(t)$. The HFHT approximation becomes valid when the field frequency $\omega$ is high compared with the typical frequency of electron motions within the molecule. The electrons then feel a time-averaged effective attractive potential, termed the “dressed potential,” given by

$$V_0(r, \alpha_0) = -\frac{Z}{2\pi} \int_0^{2\pi} \frac{d\Omega}{|r + \alpha(\Omega/\omega)|},$$

(3)

where $\Omega = \omega t$ and the time average extends over one period of the laser field.

For a homonuclear diatomic molecule with $N$-electrons and internuclear separation $R$, the Schrödinger equation in the KH frame is given by

$$\sum_{j=1}^{N} \left[\frac{1}{2}\mu_j^2 + V_0\left(r_j - \frac{R}{2}, \alpha_0\right) + V_0\left(r_j + \frac{R}{2}, \alpha_0\right)\right] + \sum_{j=1}^{N-1} \frac{Z^2}{R^3} \Phi = e^{N}(\alpha_0, R)\Phi.$$

(4)

Since the Hamiltonian is Hermitian, the energy eigenvalues $e^{N}(\alpha_0)$ are real. The field parameters $E_0$ and $\omega$ appear only in the dressed potential and enter only via $\alpha_0$, the quiver amplitude. Of prime interest are the bond energy and detachment energy,

$$\text{BE}(N, \alpha_0, R) = e^{N}(\alpha_0, R \to \infty) - e^{N}(\alpha_0, R)$$

(5)

$$\text{DE}(N, \alpha_0, R) = e^{N-1}(\alpha_0, R) - e^{N}(\alpha_0, R).$$

(6)

As long as $\text{BE} > 0$ and $\text{DE} > 0$, the molecule remains intact and retains all of its electrons. This requires that $R > R_{\text{crit}}^\alpha$, the critical internuclear distance for which $\text{BE}=0$, and $\alpha_0 > \alpha_0^\text{crit}$, the critical quiver amplitude for which $\text{DE}=0$. The bond energy reaches maxima, denoted $\text{BE}_{\text{max}} = \text{BE}(R_{\text{max}})$, at somewhat larger distances, $R_{\text{max}} > R_{\text{crit}}$, that vary with $\alpha_0$. For quiver amplitudes exceeding the critical value, the detachment energy has its maxima for the united atom ($R=0$) but typically remains positive for a substantial range of larger $R$.

III. RESULTS AT $D=3$

Previous applications of the HFHT to diatomic molecules, using $D=3$ methodology, pertain to linear polarization of the laser field and, for $\text{H}_2^+$ and $\text{H}_2$, to various values of the angle $\theta$ between the field direction and molecular axis.

For the $\text{H}_2^+$ case, Shertzer et al. employed a Hamiltonian in the KH frame of the form

$$\left[\frac{p^2}{M} + \frac{p^2}{2\mu} + \frac{1}{R} + V_0\left(r - \frac{R}{2}, \alpha_0\right) + V_0\left(r + \frac{R}{2}, \alpha_0\right)\right]u(R, r)$$

$$= Eu(R, r),$$

(7)

where $p$ and $\mu$ are the electronic and nuclear momenta, $\mu = 2mM/(m+2M)$ with $m$ and $M$ the electron and proton masses, and $V_0^\alpha$ denote the dressed potential terms of our

Eq. (3). The eigenvalues, determined by a finite element method, demonstrated that for the ground electronic state, as the field intensity is increased the molecular axis aligns more closely with the field direction (at $\theta=0^\circ$ and $180^\circ$) and the maximum bond energy $\text{BE}(R_{\text{max}})$ increases steadily (from 2.6 eV at $\alpha_0=0$ to 3.3 eV at $\alpha_0=5$). As emphasized by Zuo and Bandrauk in a similar treatment of $\text{H}_2$, the increase in the bond energy with field strength is a generic effect. This accords with Eq. (3), which makes it evident that the attractive potential energy is increased by the electric field.

For $\text{H}_2$, two approaches have been developed that enable HFHT calculations to be made using standard quantum chemistry programs. One, employed by Perez del Valle et al., uses a multipole expansion of the dressed electron-nucleus interactions $V_0^\alpha$ of Eq. (4) to determine equivalent point charge distributions. The problem is then in effect reduced to electrons interacting with several quasinuclei having fractional charges. Applied to $\text{H}_2$, for $\alpha_0<2$, using just four quasinuclei gave good accuracy for the dressed potentials. The other method, developed by Nguyen and Nguyen-Dang, modifies molecular orbitals to incorporate dressed potentials in a fashion that permits use of the GAUSSIAN 94 package. This method enabled calculations for much larger values of $\alpha_0$, but required close analysis of methodology and use of relatively large basis sets. The chief focus was on the form of the charge distributions, particularly the dichotomous clustering of electron charge in the KH frame that occurs near the endpoints of the quiver oscillations of the nuclei. Neither of these studies discussed the enhancement of chemical bonding and stabilization against ionization of $\text{H}_2^+$, our prime interest.

For $\text{He}_2$ a thorough analysis of laser induced chemical bonding in the HFHT regime was made by Tomokazu and Someda. They carried out extensive SCF-MO calculations.
and found that as the quiver amplitude grows, \( BE(R_{\text{max}}) \) increases dramatically (from zero at \( \alpha_0 = 0 - 6 \) eV at \( \alpha_0 = 1 \) and 13.9 eV at \( \alpha_0 = 2 \)). From the study of the laser induced deformation of the molecular orbitals, they identify \( sp \) hybridization of the component atomic orbitals as the major source of the chemical bonding.

### IV. EVALUATION AT LARGE-\( D \) LIMIT

The general procedure used in \( D \)-scaling\(^\text{20} \) generalizes the Schroedinger equation (by endowing all vectors with \( D \) Cartesian components) and introduces \( D \)-scaled distance and energy units to remove the major, generic dimensional dependence. The large-\( D \) limit then yields a pseudoclassical structure, in which the electrons are localized in the \( D \)-scaled space, at positions determined by the minimum of an effective Hamiltonian function. This function contains no differential operators, but comprises merely a centrifugal term, derived from the kinetic energy, in addition to the usual Coulombic interactions among electrons and nuclei. As the scaling has removed most of the \( D \)-dependence, the energy at the \( D \rightarrow \infty \) limit, found simply as the minimum of the effective Hamiltonian, typically offers a useful approximation to the \( D = 3 \) energy.

This procedure provides a natural means to examine electron localization in a superintense laser field. Since in the KH frame the electrons are stationary, in that frame the HFFT becomes equivalent to the \( D \rightarrow \infty \) limit, except that the electron-nuclear interactions are specified by the dressed potentials, with the nuclear charges distributed along the direction of polarization of the laser field. As we are concerned with linear polarization, it is convenient to use \( D \)-dimensional cylindrical coordinates (akin to \( x = \rho \cos \phi \), \( y = \rho \sin \phi \), and \( z \), in \( D = 3 \)).\(^\text{21} \) For a homonuclear diatomic molecule with \( N \) electrons, the corresponding HFFT version of the large-\( D \) limit effective Hamiltonian is then given by

\[
\mathcal{H} = \frac{1}{2} \sum_{i=1}^{N} \frac{1}{\rho_i} \sum_{j=1}^{N} V_0(\rho_i, \zeta_i; \alpha_0) + \sum_{i=1}^{N} \frac{1}{2} \sum_{j=1}^{N} \frac{1}{\sqrt{\rho_i^2 + \rho_j^2 + (\zeta_i - \zeta_j)^2}} + \xi / R \tag{8}
\]

Here \( \xi / R \) is the repulsion energy between the two nuclei; it retains the same form in the KH frame since the nuclei are quivering synchronously. The scale parameter \( \xi \) must usually be assigned a value less than \( Z^2 \) to avoid overweighting the nuclear repulsion in the large-\( D \) limit.\(^\text{22} \) For \( Z = 1 \), we adopted \( \xi = 2 / 3 \) to roughly match the large-\( D \) limit to \( D = 3 \) results for \( \text{H}_2^+ \) and \( \text{H}_2 \) when there is no laser field; for \( Z = 2 \), we used \( \xi = 4 \). Comparison with \( D = 3 \) results indicates that optimal values of \( \xi \) would vary with \( \alpha_0 \), but for our purpose we preferred not to indulge that refinement. Again, the \( V_0^i \) terms represent the dressed electron-nuclear interactions. As depicted in Fig. 1, we put the two nuclei in the \( xz \) plane with the origin midway between them and chose the laser polarization direction as the \( z \)-axis. The dressed potential terms then take the form

<table>
<thead>
<tr>
<th>( \alpha_0 ) (a.u.)</th>
<th>( D )</th>
<th>( \theta = 0^\circ )</th>
<th>( \theta = 90^\circ )</th>
<th>( \theta = 0^\circ )</th>
<th>( \theta = 90^\circ )</th>
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</thead>
<tbody>
<tr>
<td>( R_{\text{crit}} ) (a.u.)</td>
<td>3</td>
<td>0.42</td>
<td>1.2</td>
<td>1.4</td>
<td>1.5</td>
</tr>
<tr>
<td>( \approx )</td>
<td>0.49</td>
<td>0.83</td>
<td>0.98</td>
<td>1.2</td>
<td>1.4</td>
</tr>
<tr>
<td>( R_{\text{max}} ) (a.u.)</td>
<td>3</td>
<td>0.74</td>
<td>1.8</td>
<td>2.2</td>
<td>2.7</td>
</tr>
<tr>
<td>( \approx )</td>
<td>0.96</td>
<td>1.6</td>
<td>2.0</td>
<td>2.5</td>
<td>3.6</td>
</tr>
<tr>
<td>( BE_{\text{max}} ) (eV)</td>
<td>3</td>
<td>4.2</td>
<td>5.1</td>
<td>2.0</td>
<td>5.1</td>
</tr>
<tr>
<td>( \approx )</td>
<td>6.0</td>
<td>5.8</td>
<td>2.8</td>
<td>5.2</td>
<td>1.6</td>
</tr>
<tr>
<td>( DE_{\text{max}} ) (eV)</td>
<td>3</td>
<td>25</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( \approx )</td>
<td>19</td>
<td>13</td>
<td>13</td>
<td>11</td>
<td>11</td>
</tr>
</tbody>
</table>

TABLE II. Parameters for stability of \( \text{H}_2 \) in linearly polarized superintense laser fields.
Here \( \rho \) is the distance of the electron from the \( xz \) plane and \( \theta \) is the angle between the internuclear axis \( R \) and the laser polarization direction. We will consider just two special cases, with \( R \) either parallel to \((\theta=0^\circ)\) or perpendicular to \((\theta=90^\circ)\) the laser polarization. We also treat only ground electronic states.

**V. RESULTS AND DISCUSSION**

Tables I–IV present bond dissociation energies and electron detachment energies and associated parameters, as obtained from our large-\( D \) limit version of HFFT. Where available, corresponding \( D=3 \) results are included. Figures 2–4 display the bond energy curves \( BE(\alpha_0, R) \) for \( H_2^+ \), \( H_2 \), and \( He_2 \), respectively. Corresponding detachment energy curves \( DE(\alpha_0, R) \) are not shown; the quantity of chief interest, \( DE_{\text{max}} \), is included in the tables, and the \( \alpha_0 \) dependence resembles that for atoms.\(^{12}\)

For \( H_2^+ \), the bond strengths determined in the large-\( D \) limit are appreciably larger than those for \( D=3 \), but exhibit the same trends with \( R \), \( \theta \), and \( \alpha_0 \). In particular, as \( \alpha_0 \) increases, the bond dissociation energy \( BE_{\text{max}} \) increases for \( \theta =0^\circ \) but decreases for \( \theta=90^\circ \). However, as \( \alpha_0 \) increases the detachment energy for removal of the electron, \( DE_{\text{max}} \), decreases and is the same for \( \theta=0^\circ \) and \( \theta=90^\circ \). As seen in the right hand panels of Fig. 2, when scaled to \( BE_{\text{max}} \) and \( R_{\text{max}} \), the shape of the \( BE(R) \) curves for \( D \rightarrow \infty \) is similar to those for \( D=3 \) near \( R_{\text{max}} \), but approaches the dissociation asymptote more slowly at large \( R \), especially for \( \theta=90^\circ \).

For \( H_2 \), the large-\( D \) limit gives results generally like those for \( D=3 \) and displays the same trends as seen for \( H_2^+ \). Thus, again the bond strength, \( BE_{\text{max}} \), is substantially higher for \( \theta=0^\circ \) than for \( \theta=90^\circ \), whereas the detachment energy does not change as \( \theta \) is varied.

For \( He_2 \), the large-\( D \) results likewise resemble the much less extensive results available for \( D=3 \). For \( \theta=0^\circ \), the laser induced chemical bond strength for \( He_2 \) increases strongly with the quiver amplitude, much more strongly than for \( H_2 \); as evident from Eq. (9), this is a direct consequence of the increased number and magnitude of the dressed electron-nucleus interactions. The scaled \( BE(R) \) curves for the large-\( D \) limit are in this case very close to those for \( D=3 \). For \( \theta=90^\circ \), only the large-\( D \) results are available at present, but indicate that increase in \( \alpha_0 \) does not induce chemical bonding but rather augments repulsive interaction. The differences in \( BE \) between \( \theta=0^\circ \) and \( 90^\circ \) seen for \( H_2^+ \) and \( H_2 \) thus for \( He_2 \) become much more pronounced. Again, \( DE_{\text{max}} \) does not depend on \( \theta \) and decreases as \( \alpha_0 \) is increased.

Figure 5 summarizes the trends in bond energy. From the perspective of the KH frame, a heuristic explanation for the major difference between \( \theta=0^\circ \) and \( \theta=90^\circ \) is evident from Fig. 1. For linear polarization, the quiver oscillation spreads the charge of each nucleus over \( 2\alpha_0 \) but concentrates it near the endpoints of the oscillation. The consequent enhance-

### Table III. Parameters for stability of \( He_2 \) in linearly polarized superintense laser fields. For \( \theta=90^\circ \), the negative \( BE_{\text{max}} \) values given pertain to repulsive energies at \( R_{\text{max}} \).

<table>
<thead>
<tr>
<th>( \alpha_0 ) (a.u.)</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>5.0</th>
<th>7.0</th>
<th>9.0</th>
<th>11.0</th>
</tr>
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<td>( R_{\text{crit}} ) (a.u.)</td>
<td>2.0</td>
<td>3.0</td>
<td>4.0</td>
<td>6.0</td>
<td>9.0</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>( R_{\text{max}} ) (a.u.)</td>
<td>2.0</td>
<td>3.0</td>
<td>4.0</td>
<td>6.0</td>
<td>9.0</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>( BE_{\text{max}} ) (eV)</td>
<td>5.7</td>
<td>5.9</td>
<td>6.0</td>
<td>5.1</td>
<td>4.4</td>
<td>3.9</td>
<td>3.4</td>
</tr>
<tr>
<td>( DE_{\text{max}} ) (eV)</td>
<td>0.67</td>
<td>1.3</td>
<td>1.7</td>
<td>2.0</td>
<td>2.0</td>
<td>1.9</td>
<td>1.8</td>
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### Table IV. Parameters for stability of \( H_2^+ \) in in linearly polarized superintense laser fields for \( \theta=0^\circ \) at large-\( D \) limit.

<table>
<thead>
<tr>
<th>( \alpha_0 ) (a.u.)</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>4.0</th>
<th>6.0</th>
<th>8.0</th>
<th>10.0</th>
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<tbody>
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<td>( R_{\text{crit}} ) (a.u.)</td>
<td>2.0</td>
<td>2.1</td>
<td>2.2</td>
<td>1.9</td>
<td>2.0</td>
<td>2.4</td>
<td>2.7</td>
</tr>
<tr>
<td>( R_{\text{max}} ) (a.u.)</td>
<td>2.3</td>
<td>3.0</td>
<td>3.6</td>
<td>6.0</td>
<td>9.3</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>( BE_{\text{max}} ) (eV)</td>
<td>5.7</td>
<td>5.9</td>
<td>6.0</td>
<td>5.1</td>
<td>4.4</td>
<td>3.9</td>
<td>3.4</td>
</tr>
<tr>
<td>( DE_{\text{max}} ) (eV)</td>
<td>0.67</td>
<td>1.3</td>
<td>1.7</td>
<td>2.0</td>
<td>2.0</td>
<td>1.9</td>
<td>1.8</td>
</tr>
</tbody>
</table>
ment of electron density near those endpoints, termed “di-
chotomy,” is a characteristic feature of the HFFT regime.23
For a homonuclear diatomic molecule, the effect roughly
amounts to splitting the positive charges among four quasi-
nuclei, each with charge $Z/2$ and arrayed in pairs, the quasi-
nuclei within each pair separated by $2/\alpha_0$ and each pair
aligned with the laser polarization, as indicated by the “lines
of charge” in Fig. 1. When the internuclear axis $R$ between
midpoints of the quasipairs is parallel to the laser polariza-
tion ($\theta=0^\circ$), the dichotomy effect fosters an increase in
electron density between the nuclei and thus strengthens chemi-
cal bonding.14 When $R$ is perpendicular to the polarization
($\theta=90^\circ$), the dichotomy draws electron density away from
the region between the nuclei and weakens bonding.

For $H_2^+$, our large-$D$ limit results (obtained only for $\theta$
$=0^\circ$) are presented in Figs. 6 and 7 and Table IV; no $D=3$
results are available for comparison. In our previous work on
atomic anions,12 we found that for $H^-$ in the HFFT regime
the critical quiver amplitude is $\alpha_0^{\text{crit}} = 3$ a.u. Thus, unless $\alpha_0$
$> 3$ a.u., an electron cannot attach to a hydrogen atom in a
laser field. An electron can attach to a hydrogen molecule
even when $\alpha_0 < 3$ a.u. but then it can remain attached (i.e.,
$\text{DE} > 0$) only for a finite range of the internuclear distance $R$,
as seen in Fig. 6. For this range, as \( \alpha_0 \) increases \( \text{BE}_{\text{max}} \) remains nearly constant whereas \( \text{DE}_{\text{max}} \) increases strongly. When \( \alpha_0 > 3 \) a.u., since \( \text{H}^- \) is then always stable, the \( \text{H}_2^- \) anion remains stable for any \( R > R^\text{crit} \), as seen in Fig. 7. In this range, as \( \alpha_0 \) increases \( \text{BE}_{\text{max}} \) decreases whereas \( \text{DE}_{\text{max}} \) remains nearly constant.

This contrast between electron attachment for quiver amplitudes below and above 3 a.u. is further illustrated in Fig. 8. The fixed electron locations (“Lewis structure”) attained at the \( D \to \infty \) limit are shown for \( \alpha_0=2.5 \) (at left) and \( \alpha_0=8 \) (at right) and a wide range of internuclear distances. In both cases, the electron distribution broadens as either the quiver amplitude or \( R \) increases. The \( \rho \)-coordinates of the outermost electrons change only modestly with changes in \( R \), whereas \( \rho \) for the central electron varies markedly. Likewise, the \( z \)-coordinates of the outermost electrons remain close to \( \pm (\alpha_0+R/2) \) except when the central electron shifts away from \( z=0 \), thus disrupting the symmetrical configuration. Again, these features track directly changes in the form of the dressed potentials of Eq. (9), influenced by the dichotomy effect.

VI. CONCLUSIONS

In view of the computational simplicity of the large-\( D \) limit version of HFFT, it is gratifying that it gives modest accuracy when compared with conventional \( D=3 \) methods.
the oscillation frequency of the radiation field, smaller.\textsuperscript{16,17}

FIG. 8. (Color online) Variation in electronic structures (Lewis structures) at large-$D$ limit for $H_2$ in linearly polarized ($\theta=0^\circ$) superintense laser fields for $\alpha_0=2.5$ (panels at left) and 8 a.u. (panels at right) and various bond lengths $R$. Positions of the protons (black dots) are fixed along the $z$-axis and the electrons (red dots) are in the $(p-z)$ plane. As seen in Fig. 5, when $\alpha_0<3$, the extra electron remains attached only for a finite range of $R$ (thus is missing in the $R=14$ panel at left). In contrast, when $\alpha_0>3$, as seen in Fig. 6, all three electrons remain attached even as $R$ grows very large (panels at right).

At present, this appears adequate to assess prospects for experimental tests of the predicted stabilization, both with respect to electron detachment and enhancement of chemical bonding. The HFFT approximation itself is subject to validity criteria\textsuperscript{24} that in practice likely introduce uncertainties larger than disparities between the large-$D$ and $D=3$ results.

The HFFT criteria impose upper and lower bounds on $\omega$, the oscillation frequency of the radiation field,

$$137/\alpha_0 \gg \omega \gg [e^{(0)}(\alpha_0, R)]^d. \tag{10}$$

The upper bound is required to ensure that both the dipole approximation holds (radiation wavelength large compared with $2\alpha_0$) and a nonrelativistic treatment suffices (maximum quiver speed much lower than speed of light). The lower bound requires that the field must oscillate much faster than electron motion within the molecule. This is specified by an average excitation energy in the field,\textsuperscript{13} usually estimated by the energy eigenvalue that appears on the right hand side of Eq. (4). For small $\alpha_0$, this is typically comparable to the field-free ionization energy, but for large $\alpha_0$ is often much smaller.\textsuperscript{16,17}

Here we are concerned chiefly with small $\alpha_0$, and the lower bound in Eq. (10) is of the order of 1 a.u. Even to barely match this bound with $\omega=1$ a.u. would require a laser near the frontier of current technology, delivering photons with energy of $\sim 27$ eV and wavelength of $\sim 50$ nm; for $\alpha_0=1$, the beam flux needed, $I=\alpha_0^2 \omega^4$, would be $I\sim 1$ a.u.$=3.5 \times 10^{16}$ W/cm$^2$. Harmonic generation techniques are being pursued to develop such short-wavelength lasers.\textsuperscript{25} Although the lower bound can be reduced by employing larger quiver amplitudes, for a given that would also push down the upper bound and increase the required intensity. Thus, experimentally, at present both bounds of Eq. (10) can rarely be fulfilled, certainly not with “much larger” inequalities. From this perspective, the HFFT has the status of an idealized theoretical limit that can offer only a rough guide to design and analysis of experiments.

Illustrative of the utility of our simple large-$D$ version of HFFT as a qualitative guide is the evidence (Figs. 4 and 5) for the strong variation in laser induced bonding with the angle $\theta$ between the internuclear axis and the polarization of the laser field. This seems likely to be a robust property observable even if the constraints of Eq. (10) are not fulfilled. Yasuike and Someda have advocated a molecular beam experiment, colliding pairs of He atoms in an intense laser field.\textsuperscript{14} The angular distribution of scattering would reveal the presence of an attractive intermolecular potential well and its dependence on laser polarization and the quiver amplitude. Such experiments, exploiting strong laser fields to steer electron paths within collision partners, would open a new route to exploring chemical stereodynamics.

ACKNOWLEDGMENTS

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